

Power System Market Implementation in a  
Deregulated Environment

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To Maria Teresa, who has been a constant support throughout all my Ph.D. studies. Without her help, I wouldn't have been able to achieve my goals. I would like to thank her for that and so much more.

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## ABSTRACT

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The opening of the power system markets (also known as deregulation) gives rise to issues never seen before by this industry. One of the most important is the control of information. Information that used to be *common knowledge* is now kept private by the new agents of the system (generator companies, distribution companies, etc.).

Data such as the generator cost functions are now known only by the owning companies, defining a new system consisting of a group of independent firms seeking the maximization of their own profit.

There have been many proposals to organize the new market in an economically efficient manner. Nevertheless, the uniqueness of the electric power system has prevented the development of such a market.

This thesis evaluates the most common proposals using simulations in an auction setting. In addition a new methodology is proposed based on mechanism design, a common technique in economics, that solves some of the practical problems of the power system market (such as the management of limited transmission capacity). In this methodology, when each company acts in its best interest, the outcome is efficient in spite of the information problem cited above. This new methodology, along with the existing methodologies, are tested using simulation and analyzed to create a clear comparison of benefits and disadvantages.

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## CHAPTER 1 INTRODUCTION

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The application of competitive markets has been successful in several sectors of the U.S. economy. The application of competition in electric power markets has not been completed. What makes the electric power market different is the presence of a transmission system, where the flow of power cannot be easily controlled. In addition, the scarcity of transmission capacity leads to congestion or potential overloading. In spite of the efforts made to create an open, free market, electric power companies still have to cope with the transmission congestion problem.

Many different methods to structure and operate the electric power markets have been proposed. This thesis presents a new algorithm to solve the congestion problem using a technique called mechanism design. Using this technique and taking the transmission network constraints seriously, one can design a mechanism such that, when each participant acts in its own best interest, the outcome of the daily operation of the electric power market is economically efficient (achieving optimum economic dispatch).

### **1.1 Regulated Power Systems**

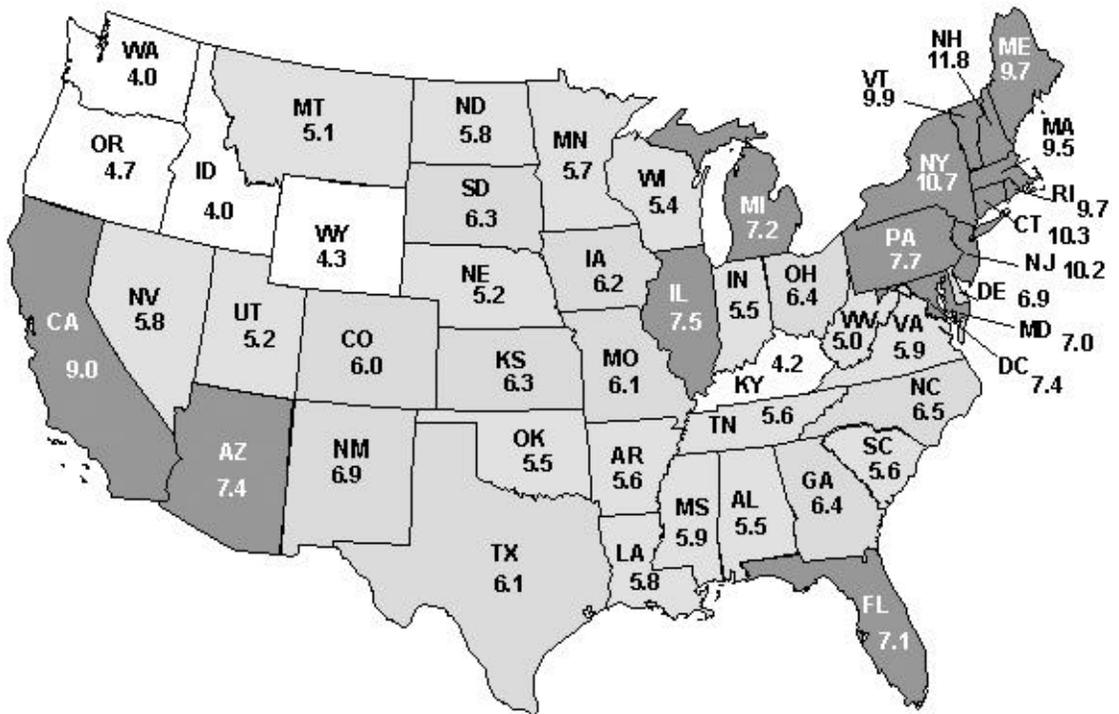
Governments foresaw the importance of providing reliable electricity at a reasonable price, and they assigned electric power production and distribution to public-

ownership or private-highly regulated monopolies. These monopolies include the three main activities of the electric industry: generation, transmission and distribution. In the US, electric utilities had the monopoly right to sell energy within a specific geographic area. The utility, on the other hand, also had the obligation to supply energy to all customers in its area. Either government agencies or public utility commissions would dictate the price of electricity, guaranteeing the utility a repayment of its costs (including future investments and operational costs) and a reasonable return on its investment.

## **1.2 Competitive Power System Markets**

After successfully deregulating other network-based industries (such as the natural gas and telephones), seeing the successful experiences overseas, and experiencing the high costs of producing energy during the 1970's oil embargo, economists were convinced that a shift to more efficient schemes of electricity production were possible and needed.

Pressure for change also came from industries in high-price areas (such as New England and California) that wanted to be able to purchase energy from other states where the prices were lower. The following figure shows the estimated revenue per kWh that utilities received (paid by industrial and consumer customers) by state in 1998. (Source: U.S. Energy Information Administration, [Energ98].)



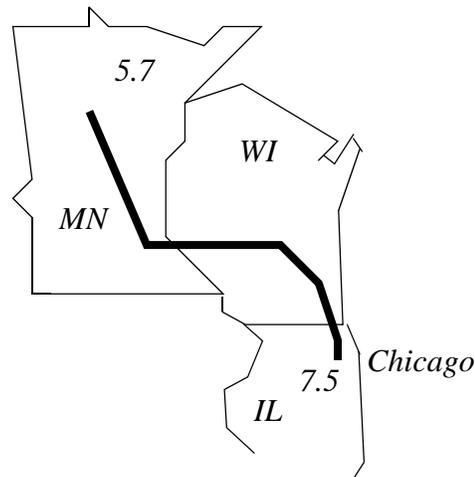
**Figure 1-1: Estimated Average Revenue in Cents per kWh for All Sectors of U.S. Electric Utilities by State, 1998**

In the figure all the states with low cost energy (0 to 4.99 [cents/kWh]) are shown in white, the moderate cost energy (5 to 6.99 [cents/kWh]) in gray, and with expensive energy (over 7 [cents/kWh]) in dark gray. This figure shows that the areas with more advanced deregulated markets (New England and California) have a higher price, demonstrating the pressure from the industrial sector to deregulate the market so they will have access to cheaper prices from other areas of the country (such as Minnesota with estimated price of only 5.7 [cents/kWh]).

The idea of deregulation is good, but not all of the electric system is suitable for such a change. Distribution and transmission are *natural monopolies* that invalidate them as participants in an open competitive market (for more details see in Chapter 3). This leaves generation as the only sector suitable for a competitive market. But this does not mean that distribution and transmission would be untouched. Competition can be established in generation, but only if the necessary changes are introduced in distribution and transmission to allow and encourage a competitive generation market.

The first notion of deregulation in the U.S. appeared in the Public Utilities Regulatory Policies Act (PURPA) in 1978. This document encouraged the purchase of energy from different power producers. More than a decade later, the Federal Energy Regulatory Commission (FERC) issued the Energy Power Act (1992) establishing the right of open access to the transmission network to any entity that does not own transmission. The idea was to multiply the number of potential producers that the customer could buy from, which is essential for the success of a competitive market. But even with the good intentions of the government agencies, the transition to an open market has proven to be very difficult. The main reason for this is the existence of a limited transmission network that makes the power system market different. In the case of Figure 1-2 a generator company in Minnesota desires to take advantage of the business opportunity of selling its power in the Chicago area. By doing so the company can get an additional 1.8 [cents/kWh] (this can significantly increase the company's profit). Nevertheless, the barrier between the low cost energy from Minnesota and Chicago is the

transmission interface between them. Most of the time the transmission interface operates at its maximum, not allowing any more transfer in that direction.



**Figure 1-2: Transmission Lines Linking Minnesota and Chicago and Prices in Both Areas [cents/kWh]**

Theoretically, if the open market spreads to all the states the energy prices should level out (it would actually rise in low-price states such as Minnesota and decrease in high-price states such as Illinois). Nevertheless, the existence of congestion in the interfaces (such as in Minnesota-Illinois) will always keep the low-price power from getting to the high-price areas. In the market, this will reduce the number of generator companies that can offer their power in high-price areas to only local generators and a small fraction of generators from other areas, allowing the local generators to raise their prices due to the lack of competition.

One alternative to solve the congestion problem is to construct new lines, but this is not

as easy as it may seem; transmission lines are expensive and they are the target of a variety of groups who wish to block their construction. The other alternative is to create a mechanism of congestion management that would allow a competitive market to operate even with limited transmission capacity, leading to an efficient use of both the transmission and generation resources.

### **1.3 Transition from Regulated to Competitive Power System Markets**

Electrical engineers knew how to solve the economic dispatch to minimize costs in a highly regulated environment. The basic difficulty under deregulation is that underneath the economic dispatch lies the assumption that the key information about the participants in the market (heat rate curves and fuel costs of generators, willingness to pay of consumers, etc.). But in a deregulated environment, it cannot be taken for granted that a coordination entity (pool operator, etc.) is going to be given this key information from the market participants.

### **1.4 The California ISO Case**

This section includes a brief description of the congestion management problem that is being observed in California.

### 1.4.1 California ISO System

Deregulation in California resulted in the establishment of the California ISO (Independent System Operator). This ISO tries to cope with congestion by dividing its system into zones. The idea was to limit the congestion to only the interfaces among zones. In the near future the ISO will allow every agent to buy Firm Transmission Rights to use the interfaces between zones; these rights will be purchased in a secondary market. The effectiveness of this mechanism is yet to be seen (it will start in 2000).

The designers of the ISO thought that, after defining the interfaces, the transmission systems within the zones would be congestion-free and the competitive market would succeed.

The first problems appeared when the ISO found congestion inside a zone (due to a fault on a transmission line). Given this congestion the ISO accepts real-time energy bids on both sides of the congested line for the minimum necessary generation adjustment to solve the problem. (All the quotes are from [Ferc00].)

*The ISO accepts all intrazonal transmission schedules without first determining if all of the schedules are feasible. If all of the schedules are not feasible, the ISO will accept real-time energy bids that relieve the constraint.*

The ISO noticed that generator companies, realizing the lack of alternatives for the

ISO, started overcharging for their power used to relieve the congestion. (If the ISO has to buy from a company, then this company can charge any price it wants.)

*In one example cited by the ISO, the unconstrained adjustment bid on August 1, 1999, was 45 mills/kWh which was accepted when a transmission line went out of service. The next hour, the adjustment bid from this same resource jumped first to 78 mills/kWh and then to 227 mills/kWh. In another example, on October 28, 1999, following the loss of transmission lines, the adjustment bids increased to 710 mills/kWh.*

#### **1.4.2 Reactions to the Problem**

The California ISO has tried to solve this congestion problem by asking FERC (Federal Energy Regulatory Commission) permission to extend its ability to obligate generators to provide power in cases of RMR (Reliability-Must-Run) to the cases lacking market competition. (In the RMR case the ISO only gives a cost-based payment to the generators.)

*There is no dispute that the ISO currently has the authority to direct any Participating Generator to change its dispatch when the ISO deems it necessary to protect system reliability.*

The generator companies, aware of the profit they are obtaining, are fighting the new proposal of the ISO arguing that high prices are needed to give new participant the correct location signal to install their units. In particular they relate these high prices to scarcity and not to market power. (From an economic approach, in the case of a

competitive market, high prices mean scarcity of the product, but in the case of non-competitive market they are just a reflection of market power.)

FERC recognized the failure of the congestion management scheme of the California ISO and attribute it to the lack of a practical experience with the system before it was established. Nevertheless, it does not support either of the two extremes (California ISO or generator companies) and called for a new beginning, rejecting the idea of just patching the system.

Clearly the California ISO is experiencing the problems of congestion management. This thesis gives a foundation for analyzing these problems and solves some of the most common difficulties of power system markets.

## **1.5 Bibliography Background**

In the current bibliography there are two main lines of development to deal with transmission pricing and congestion management. The first one is the “tradeable physical rights” that includes the concept of a right to inject power at a given bus and withdraw that power at another bus of the network. In practice this system is based on the periodic calculation of the Available Transmission Capacity using a series of assumptions about the load, reliability and system conditions in general. The major difficulty with this approach is that after all the assumptions that are made, the usefulness of the results from the Available Transmission Capacity algorithm are not

clear.

The other line of development is that of “nodal pricing” by Fred Schweppe ([Schwe88]), and later extended by Hogan ([Hogan93] and [Harvey96]). The idea of this approach is to run an auction market for energy and use the bids to derive a “least cost” merit order of generators and clearing prices. On the supply side generators quote their prices for energy, and on the demand side, customers quote their willingness to pay. After this process is done, the network operators enter the bids into an optimization program (that includes the transmission capacity limits) and assign quantities of electricity to be produced by each participant.

This thesis focuses on the second approach, “nodal pricing”, giving an original solution to the congestion management problem.

## **1.6 Thesis Structure**

This thesis is organized as follows. This chapter gives an introduction to the problem of congestion management in the power system marketplace. Chapter 2 describes power engineering concepts such as economic dispatch, power flow and optimal power flow. Chapter 3 presents the key elements from economics that support the ideas presented. These concepts include: offer and demand in electricity markets, efficiency, individual rationality and an introduction to the basic ideas of mechanism design. Next, Chapter 4 presents some of the current methodologies to structure and operate the

market using payments to the agents as the cornerstone of the discussion. Among the methodologies included are bus marginal cost payment and the transmission congestion contract. To complement the methodologies shown in Chapter 4, Chapter 5 describes the incentive compatible mechanism which is the central proposal of this thesis as a solution to the congestion management problem.

In order to show the advantages and disadvantages of the current methodologies (including the incentive compatible mechanism) this thesis uses simulations. Chapter 6 describes the assumptions that are considered for the simulations and then describes the different techniques used to simulate electricity markets. Using simulations of the agent bid algorithm, Chapter 7 demonstrates that the incentive compatible mechanism works using a comparison with traditional congestion management. The systems simulated in this chapter are a 2-bus example, the IEEE-14-bus system and a 6-bus system. In Chapter 8 the static simulation are extended to a multi-period technique using a 3-bus system. In this chapter the incentive compatible mechanism is compared with each of the current methodologies described in Chapter 4. Chapter 9 concludes and presents the future challenges that can be followed in order to continue with this line of development.

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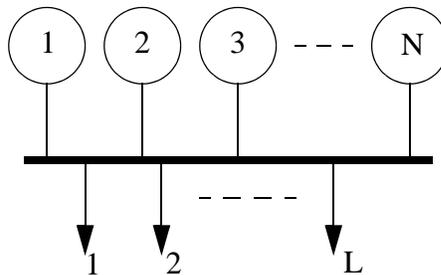
## CHAPTER 2 POWER ENGINEERING CONCEPTS

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This chapter describes the more important concepts of power engineering supporting this thesis. Although, most of this material is not new for power system engineers, they may find some of the insights interesting.

### 2.1 Economic Dispatch

An economic dispatch algorithm solves the problem of allocating the available generator output to serve a given load. Simply enough, [Wood96] describes the economic dispatch problem as a system with  $N$  generating units connected to a single bus-bar serving the sum of  $L$  individual loads.



**Figure 2-1: Economic Dispatch**

The economic dispatch constraint is the application of the conservation of energy at a

bus:

$$\sum_{j=1}^L P_{load_j} - \sum_{i=1}^N P_{gen_i} = 0 \quad (2.1.1)$$

In the classical power engineering bibliography [Wood96] the objective function is identified with the minimization of the sum of the generator costs. But, in a more general way, the objective function of an economic dispatch problem corresponds to the maximization of the benefit to consumers. This is due to the primary and essential objective of a power system which is to serve consumers and the objective function should reflect the benefit that electricity brings to them. Therefore, a more general formulation for the optimal dispatch will be:

$$Max \sum_{j=1}^L \alpha_j U_j(P_{load_j}) \quad (2.1.2)$$

Subject to,

$$\sum_{j=1}^L P_{load_j} = \sum_{i=1}^N P_{gen_i} \quad (2.1.3)$$

where:

$P_{load_j}$  - amount of power assigned to consumer j.

$\alpha_j$  - non-negative number representing the weight of consumer j's utility in the overall objective function. (Since utility functions only represent personal preferences they need to be weighted in order to be added.)

$U_j(P_{load_j})$  - utility of load j as a function of  $P_{load_j}$ . The utility function measures the overall benefit to the consumer for a given consumption  $P_{load_j}$  (this concept is going to be described in Chapter 3).

$P_{gen_i}$  - the quantity of power produced by generator i.

$N$  - the number of generators.

$L$  - the number of generators.

The equivalence between the maximization of utility and the minimization of cost (classical approach) is true if the following assumptions are accepted:

- Load is inelastic with respect to price. Therefore, the consumer will buy the same amount of power for any possible price, implying a fixed total production amount. Since the load is fixed, the utility maximization is just the minimization of the price the consumer has to pay.
- The generator company would receive a constant payment for their

production. Therefore, the only way to maximize their profit is to minimize their cost.

Using these two assumptions the problem of maximizing utility becomes equivalent to the problem of minimizing the production cost.

## **2.2 Power Flow**

What makes the electricity market so different from other network-based markets is the existence of the transmission network. This network is governed by a series of relations called the power flow equations. These equations are non-linear and in the case of large systems they are very numerous (one or two per bus), which makes the solution of a power flow a difficult problem by itself. (The eastern U.S. power flow model requires approximately 20,000 buses and results in about 40,000 simultaneous equations to be solved.)

A power flow algorithm computes the voltage magnitude and phase angle at each bus in a power system under balanced three-phase steady-state conditions. It also calculates the real and reactive power flows for all transmission lines and transformers, as well as losses in the different components in the system.

The power flow solves a system of simultaneous non-linear equations finding the system's state variables (these variables are bus voltages and phase angles for an AC

power flow and phase angles for a DC power flow [Wood96]). System components are transmission lines, generators, loads, transformers, phase shifters, shunt reactive support components and FACTS (Flexible AC Transmission Systems) devices such as static converters, series compensators, etc.

### 2.3 Power Flow Formulation

The starting point in the power-flow formulation consists of applying the conservation of energy law at all nodes (Kirchhoff's node law). At any given moment, the sum of energy coming into the node must equal the energy leaving. It also can be written in terms of power as:

$$S_k = \sum_i^n S_{ki} \quad (2.3.4)$$

Where  $S_k$  is a complex value  $P_k + jQ_k$  and  $S_{ki}$  is the flow from bus  $i$  to bus  $k$ .

Using Ohm's law the power flow problem can be written in complex form as:

$$S_k = V_k I_k^* = V_k \sum_i^n I_{ki}^* = V_k \sum_{i \neq k}^n (Y_{ki} \cdot V_i)^* \quad (2.3.5)$$

where:

$Y_{ik}$  is  $i$ - $k$  term of the  $Y$  matrix,  $V$  is complex voltage and  $I$  is complex current, where,

$$V = |V|\angle\delta \quad (2.3.6)$$

The  $Y$  matrix gives the admittance among the buses. The rules for forming the  $Y$  matrix are:

If a line exists from  $i$  to  $j$ ,

$$Y_{ij} = -y_{ij} \quad (2.3.7)$$

and

$$Y_{ii} = \sum_j y_{ij} + y_{ig} \quad (2.3.8)$$

$j$  over all lines connected to  $i$ . Where  $-y_{ij}$  is the admittance between buses  $i$  and  $j$ , and  $y_{ig}$  is the admittance between bus  $i$  and the ground reference.

## 2.4 Power Flow Methods

Different solution methods have been applied to solve the power-flow problem. Some of these techniques are Gauss-Seidel, Newton-Raphson, Decoupled power flow, DC power flow and Continuation power flow. For a precise solution the most used method is the Newton-Raphson power flow. But when a fast and good approximation is

needed, the DC method is the most widely used. In the following two subsections these two methodologies are described.

### 2.4.1 The Newton-Raphson Method

A set of nonlinear algebraic equations in matrix format given by:

$$F(X) = Y \quad (2.4.9)$$

where  $Y \wedge X \in \mathfrak{R}^n$ . Given  $Y$  and  $F(X)$ , a solution for  $X$  can be found. Expanded  $F(X)$  as a Taylor's series:

$$Y = F(X_0) + \left. \frac{dF}{dX} \right|_{X=X_0} (X - X_0) + \dots \quad (2.4.10)$$

neglecting the higher order terms and solving for  $X$ ,

$$X = X_0 + \left[ \left. \frac{dF}{dX} \right|_{X=X_0} \right]^{-1} (Y - F(X_0)) \quad (2.4.11)$$

The Newton-Raphson method replaces  $X_0$  by  $X$  the old value  $X(i)$  and by the new value  $X(i+1)$  in eq. (2.4.11).

Thus

$$X(i+1) = X(i) + J^{-1}(i)[Y - F(X(i))] \quad (2.4.12)$$

where

$$J(i) = \left. \frac{dF}{dX} \right|_{X=X_0} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad (2.4.13)$$

The  $n \times n$  matrix  $J(i)$ , whose elements are the partial derivatives shown in eq. (2.4.13), is called the Jacobian matrix.

Applying the Newton-Raphson method to the power flow solution, the equations can be written as,

$$X = \begin{bmatrix} \delta \\ V \end{bmatrix}; \quad y = \begin{bmatrix} P \\ Q \end{bmatrix}; \quad F(X) = \begin{bmatrix} P \left( \begin{matrix} \delta \\ V \end{matrix} \right) \\ Q \left( \begin{matrix} \delta \\ V \end{matrix} \right) \end{bmatrix} \quad (2.4.14)$$

where all terms  $V$ ,  $P$ ,  $Q$  are vectors in per-unit and  $\delta$  are vectors in radians.

Thus,

$$P_i + jQ_i = V_i \sum_{k=1}^N Y_{ik}^* V_k^* \quad (2.4.15)$$

This can be expanded to:

$$P_i + jQ_i = \sum_{k=1}^N |V_i| |V_k| (G_{ik} - jB_{ik}) e^{j(\theta_i - \theta_k)} \quad (2.4.16)$$

$$P_i + jQ_i = \sum_{k=1}^N \{ |V_i| |V_k| [G_{ik} \cos(\theta_i - \theta_k) + jB_{ik} \sin(\theta_i - \theta_k)] \} \quad (2.4.17)$$

$$+ j |V_i| |V_k| [G_{ik} \sin(\theta_i - \theta_k) - jB_{ik} \cos(\theta_i - \theta_k)] \}$$

where:

$\theta_i, \theta_j$  - the phase angle at buses i and k respectively.

$|V_i|, |V_k|$  - the bus voltage magnitudes, respectively.

$G_{ik} + j B_{ik} = Y_{ik}$  - the  $i$ - $k$  term in the  $Y$  matrix of the power system  $D/|V_i|/|V_i|$  has been

used in practice instead of  $\Delta|V_i|$ ; this simplify the equations. The derivatives are:

$$\frac{\partial P_i}{\partial \theta_k} = |V_i||V_k|[G_{ik}\sin(\theta_i - \theta_k) - jB_{ik}\cos(\theta_i - \theta_k)] \quad (2.4.18)$$

$$\frac{\partial P_i}{\left(\frac{\partial |V_k|}{|V_k|}\right)} = |V_i||V_k|[G_{ik}\cos(\theta_i - \theta_k) + jB_{ik}\sin(\theta_i - \theta_k)]$$

$$\frac{\partial Q_i}{\partial \theta_k} = -|V_i||V_k|[G_{ik}\sin(\theta_i - \theta_k) + jB_{ik}\cos(\theta_i - \theta_k)]$$

$$\frac{\partial Q_i}{\left(\frac{\partial |V_k|}{|V_k|}\right)} = |V_i||V_k|[G_{ik}\sin(\theta_i - \theta_k) - jB_{ik}\cos(\theta_i - \theta_k)]$$

For  $i = k$

$$\frac{\partial P_i}{\partial \theta_k} = -Q_i - B_{ii}V_i^2 \quad (2.4.19)$$

$$\frac{\partial P_i}{\left(\frac{\partial |V_i|}{|V_i|}\right)} = P_i + G_{ii}V_i^2$$

$$\frac{\partial Q_i}{\partial \theta_k} = P_i - G_{ii}V_i^2$$

$$\frac{\partial Q_i}{\left(\frac{\partial |V_i|}{|V_i|}\right)} = Q_i - B_{ii}V_i^2$$

For more details see [Wood96].

### 2.4.2 DC Power Flow

A very popular simplification of the power flow is to drop the equations relating  $V$  and  $Q$  and keeping only the equations relating  $P$  and  $\theta$ . This, and some additional assumptions convert a complex non-linear the power flow problem into a linearization that solves without iterating.

The list of assumptions can be obtained in [Wood96] and is applied to the Newton-Raphson formulation of subsection 2.4.1:

- Neglect the relationship between  $P_i$  and  $|V_k|$ , and between  $Q_i$  and  $\theta_k$ . This can be done because of the weak interaction of these two pairs compared to the pairs  $P_i - \theta_k$  and  $Q_i - |V_k|$ . With these assumptions the terms of the Jacobian as

seen in eq. (2.4.18) are reduced to:

$$\frac{\partial P_i}{\partial \theta_k} = |V_i||V_k|[G_{ik}\sin(\theta_i - \theta_k) - jB_{ik}\cos(\theta_i - \theta_k)] \quad (2.4.20)$$

$$\frac{\partial P_i}{\left(\frac{\partial |V_k|}{|V_k|}\right)} = 0$$

$$\frac{\partial Q_i}{\partial \theta_k} = 0$$

$$\frac{\partial Q_i}{\left(\frac{\partial |V_k|}{|V_k|}\right)} = |V_i||V_k|[G_{ik}\sin(\theta_i - \theta_k) - jB_{ik}\cos(\theta_i - \theta_k)]$$

- Approximate the cosine as unity. This assumption can be made because the cosine arguments are close to zero, allowing a neglect of the 2nd and successive terms of the Taylor approximation of the cosine.

$$\cos(\theta_i - \theta_k) \approx 1 \quad (2.4.21)$$

- Neglect the terms  $G_{ik}\sin(\theta_i - \theta_k)$ , which are relatively small with respect to  $B_{ik}$ . Also eliminate all shunt reactances to ground.

- Drop the relationship between Q and V. This can be done if the main interest of the study is the active power and the system is operating near its nominal value of voltage. Therefore the new power flow equations resulting from these

assumptions can be written as:

$$\frac{\partial P_i}{\partial \theta_k} = -|V_i||V_k|B_{ik} \quad (2.4.22)$$

- Assuming that the voltages are near 1 in p.u. allows eliminating the voltage magnitudes from eq. (2.4.22).

Finally, one can assume that  $r_{ik} \ll x_{ik}$  due to the fact that transmission lines are mostly reactive and not resistive. Therefore,

$$B_{ik} = -\frac{1}{x_{ik}} \text{ and } B_{ii} = \sum_{j \in \Phi_i} \frac{1}{x_{ik}} \quad (2.4.23)$$

for an  $i$  different from  $k$  and for the same bus. Where  $\Phi_i$  is the set of lines connected to bus  $i$ . In the case of a line existing between  $i$  and  $k$  the value of  $B_{ik}$  will be different from zero, and zero otherwise.

The linear system using all these assumption looks as follows:

$$\begin{bmatrix} P_1 \\ P_2 \\ \dots \\ P_N \end{bmatrix} = \begin{bmatrix} \sum_{j \in \Phi_1} \frac{1}{x_{1j}} & -\frac{1}{x_{12}} & \dots & -\frac{1}{x_{1N}} \\ -\frac{1}{x_{21}} & \sum_{j \in \Phi_2} \frac{1}{x_{2j}} & \dots & -\frac{1}{x_{2N}} \\ \dots & \dots & \dots & \dots \\ -\frac{1}{x_{N1}} & -\frac{1}{x_{N2}} & \dots & \sum_{j \in \Phi_N} \frac{1}{x_{Nj}} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dots \\ \theta_N \end{bmatrix} \quad (2.4.24)$$

This equation system has a linearly dependent equation that needs to be eliminated. Also one of the angles need to be set to a fixed value (typically zero) so the remaining are defined with respect to this reference.

## 2.5 Optimal Power Flow

An optimal power flow (concept first developed by Carpentier, [Carpe62]) has the starting point of an economic dispatch, plus a set of additional constraints representing the entire set of AC power flow equations.

The additional variables that are normally added to the dispatch problem are: generator voltage magnitude, transformer tap position, phase shift transformer position, switched capacitor position, DC line flow, etc. The number of variables and constraints in this problem make it difficult to solve.

This thesis uses a simplified version of the complete optimization problem that can be seen in [Wood96]. This simplification can be stated as the minimization of the total production cost subject to (i) DC power flow constraints, (ii) line thermal limits, and (iii) generation limits. The OPF formulation goes as follows:

$$\min \sum_{i=1}^{N_g} C_i(P_{gen_i}) \quad (2.5.25)$$

subject to:

(i) DC power flow equation for each bus  $i$ ,

$$\sum_{j \in \Phi_i} \frac{1}{x_{ij}} (\theta_i - \theta_j) = P_{gen_i} - P_{load_i} \quad (2.5.26)$$

(ii) line thermal limit constraint for all pairs  $i$ - $j$  corresponding to existing lines,

$$P_{line_{ij}}^{min} \leq \frac{1}{x_{ij}} (\theta_i - \theta_j) \leq P_{line_{ij}}^{max} \quad (2.5.27)$$

(iii) generation constraints for all generators  $i$ ,

$$P_{gen_i}^{min} \leq P_{gen_i} \leq P_{gen_i}^{max} \quad (2.5.28)$$

where:

$P_{gen_i}$  - the quantity of power produced by the generator at bus i.

$C_i(P_{gen_i})$  - the cost of generator i as a function of  $P_{gen_i}$ .

$N_g$  - the number of generators.

$\theta_i$  - bus i's phase angle.

$P_{load_i}$  - bus i's load.

$\Phi_i$  - the set of lines connected to bus i.

$P_{line_{ij}}^{min}, P_{line_{ij}}^{max}$  - minimum and maximum thermal capacities, respectively, for the transmission line from bus i to bus j. Usually  $P_{line_{ij}}^{min} = -P_{line_{ij}}^{max}$  indicates the same thermal limit on flow in the both directions.

$P_{gen_i}^{min}, P_{gen_i}^{max}$  - the minimum and maximum quantities that generator i can produce, respectively.

Notice that in order to apply the OPF a system *coordination entity* must know the cost functions and power limits of all generators, as well as the power limits and reactances of the transmission lines. It is also important to notice that the OPF formulation can include consumers simply adding their utility functions to the objective function. (The concept of utility function is going to be presented in Chapter 3.)

### 2.5.1 Bus Marginal Cost

A by-product of the Optimal Power Flow problem described in last section is the bus marginal cost at every bus of the system. Bus marginal costs correspond to the Lagrange multipliers associated with each of the DC power flow constraints of eq. (2.5.26).

From a mathematical point of view, the bus marginal cost of any bus  $i$  corresponds to the increment (or decrement) of the objective function (cost function of the system) for an increment (or decrement) in the demand at bus  $i$ .

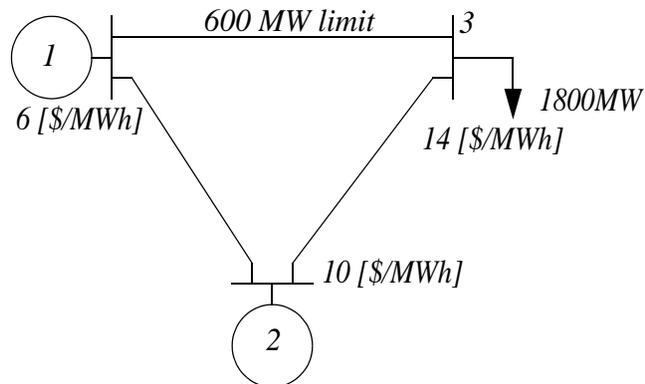
This concept can also be extended to transfers across a network. The change in the objective function for the system due to transferring one additional MW from bus  $i$  to  $j$  corresponds to the difference in the marginal cost between these two buses.

These two ideas are the basis for a very reasonable payment scheme as is going to be seen in Chapter 4.

### 2.5.2 Congestion

Bus marginal costs change in response to congestion in a network. The problem of congestion arises where a limitation in the transmission network (one or more lines reaching a limit) does not allow the most convenient energy to flow to all areas of the network. This produces a difference in the bus marginal costs between the areas separated by the congested path, so that the cost of supplying one additional MW (bus

marginal cost) in the buses affected by the congestion is going to be more than in the remaining buses.



**Figure 2-2: 3-bus System Showing Marginal Costs with Line Limit**

In the example of Figure 2-2 two scenarios can be analyzed: in the first one, there is no limit on the transmission line 1-3, generator 1 (low cost) can provide all the energy serving the load at bus 3 (1800 MW). The cost of supplying an additional MW (bus marginal cost) in all buses would be 6 [\$/MWh]. In the second scenario, the transmission line 1-3 has a 600-MW limit, this creates congestion and generator 1 will not be able to provide all the energy required by bus 3. This congestion will force load 3 to buy from generator 2 (high cost). In this case, the cost of supplying an additional MW (bus marginal cost) is different at each bus, reaching values of 6 [\$/MWh] at bus 1, 10 [\$/MWh] at bus 2, and 14 [\$/MWh] at bus 3. Notice that the existence of congestion will create different bus marginal costs.

## 2.6 Long Term vs. Short Term

The problems that regulator and utilities face in the electric power system can be divided into three levels: planning, operation and real-time operation. These three separate problems have different time horizons, objective functions and constraints.

The planning problem refers to the question: given the power system as it is, a forecast of the load and a certain financial constraint (budget) what is the optimum investment path for the next 5 or 10 year period? This may seem simple, but each element mentioned above has its own complication. For example, the load forecast is a very difficult task requiring information about the economy and population of a region. The planning team needs to be given not only a reasonable set of load scenarios, but also the location of each load.

The objective function of a utility is the maximization of the expected profits. In the regulated world, this translates to a minimization of cost of investment (build new plants, transmission lines, etc.) plus minimization of the expected cost of operation. In a regulated environment the selling price is given, so the income is fixed. Therefore, the only way to maximize profit is to minimize costs. This may seem to be an incentive to achieve efficiency, but this can be misleading because the price is set using this same cost. (In general, prices are set to provide the utility with its cost plus a certain compensation for its investment.)

The next level is the operation of the system. For this problem, the operator is given a certain power system configuration, a short term (more precise) load forecast and a set of technical constraints (generator limits, transmission limits, etc.). The horizon of the operation problem usually goes from days to hours ahead (in the case of system having a strong hydro component this horizon can be much larger). The operator needs to create a plan to follow the load (demand), warranting a level of security and reliability for the system at the minimum cost possible (mostly fuel cost). The analysis the operator performs includes: contingency analysis, transient stability studies, etc.

The last level is real-time operation. Operators here are given some guidelines on how to operate the system (from the operation level) and a more accurate short-term load forecast. Although this information is valuable for operators, they do not always follow it, because their objective is to have the system up and running. Real-time operators in different areas use a generation control system to follow the load while trying to maintain the frequency and the scheduled transfers among areas under reasonable limits. In general, the position of real-time operators are filled with people that know the system and are fast at coordinating with their neighboring operators so as to protect the system against any unexpected contingency that may occur.

After the real-time operation utilities adjust all the differences between what was agreed in the operation layer and what actually happened. This process is called settlement.

Considering this description of the activities in a power system one can locate the developments in this thesis as belonging to the operations level, with companies' investment decisions as given.

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## CHAPTER 3 ECONOMIC CONCEPTS

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This chapter briefly presents a description of the economic concepts supporting this thesis. Most of these concepts are related to the idea of preference and utility. According to Mas-Colell [Mas-C95].

*A preference is a binary relation defined on a set of alternatives, allowing the comparison of pairs of alternatives.*

For example, having an individual with a set of alternatives (an apple, a banana and a coconut), a preference will establish the binary relation of the type *at least as good as* between all the elements of the set. The idea of a utility function is to use the basic concept of preference to define a function that described such a preference with numbers.

*A utility function  $u(x)$  assigns a numerical value to each element of a set of alternatives, ranking the elements in accordance with the individual's preferences.*

In this thesis the concept of preferences of a company (either a generator or a distribution company) is defined over the set of the real numbers corresponding to the profit the company gets by subtracting the operating costs from revenues. It is understood that the preferences of a company reflect the idea that more profit is always better. Therefore, the utility function can easily be defined as the amount of profit. In

the case of an individual (a person) a utility function is more difficult to define and is discussed in subsection 3.3. Notice that, since this work considers only consumer companies (distribution companies) having thousands of consumer customers and not individual consumers, only the first definition has meaning in the remaining chapters of the thesis.

### **3.1 Efficiency**

The formalization of the concept of efficiency is due to the work of the economist and sociologist Vilfredo Pareto, [Mas-C95]. Pareto expressed the idea that an allocation of resources is efficient if there is no other feasible allocation that makes some participant better-off and no others worse-off. This definition can be applied to consumers in general, but it can also be extended to the traditional power system economic dispatch problem, in which consumers are hidden behind a constant load and the objective function is simply the minimization of the cost of generation. Notice that there is an implicit assumption that the compensation given to a generator for the power it produces is constant, so the same solution can be obtained from the minimization of the generation cost, as from the maximization of the generator profit (see subsection 2.1).

Moreover, the traditional economic dispatch is totally consistent with the concept of Pareto efficiency. For example, consider a one-bus system with two generators and a load. Assuming that both generator marginal costs increase with output. Pareto

efficiency requires that generators produce at the same marginal cost. Otherwise, one could have the generator with higher marginal cost produce less (thereby lowering its marginal cost) and the other produce more (thereby raising its marginal cost). It can be seen that such a re-allocation reduces the operating cost. However, operating at identical marginal cost is also a requirement of economic dispatch resulting from the first order condition of the minimization of the cost.

### **3.2 Generator Company Offer Curve**

This subsection shows the necessary steps to obtain an expression describing the *offer curve* of a generator company. This function gives the amount of power the generator company is willing to produce (or offer) given price as a parameter.

Any generator company is simply a firm or a company that is profit-driven. As with any firm, a generator company will always try to maximize its profit. For the sake of simplicity, this maximization of profit is decoupled in time, allowing the analysis of one period at the time (typically one hour in length).

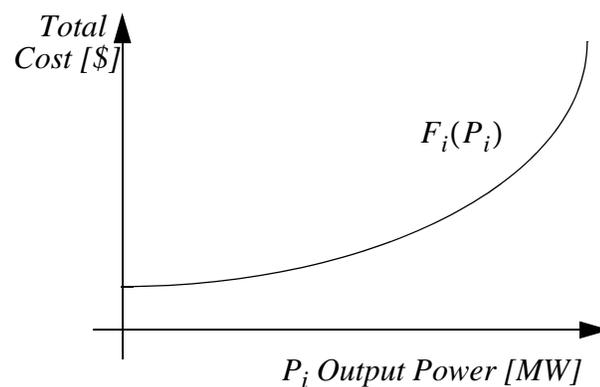
At any given time period, the profit is calculated as income minus costs. For a generating company the most important component of the profit is the payment it gets for its power. Total cost comes from investment (investment) in the facility and paying for the fuel (operation).

### 3.2.1 Technical Limits of the Offer Curve

The first and most obvious limitations of the offer curve of a power plant are its technical limits (generation minimum and maximum limits). In other words the plant will not be able to produce less than a certain minimum output (which is determined by the characteristics of the generator's boiler-turbine) and more than another amount (given by the maximum output of the plant, again set by the boiler-turbine). This is a very important point especially in the case of thermal units that only produce in a narrow band of output and not as important to the hydro unit that can vary its output a great deal without losing efficiency.

### 3.2.2 Total and Average Cost

A generation company can have one or more power plants. The cost function of each one of these can be modeled as an increasing function of the output power:



**Figure 3-1: Total Cost**

The intersection of this curve with the y-axis or fixed component reflects the cost that

does not depend on the level of output, which the company must pay even if its power production is zero (minimum amount of coal that must be fed to the boiler in order to maintain a certain temperature and pressure).

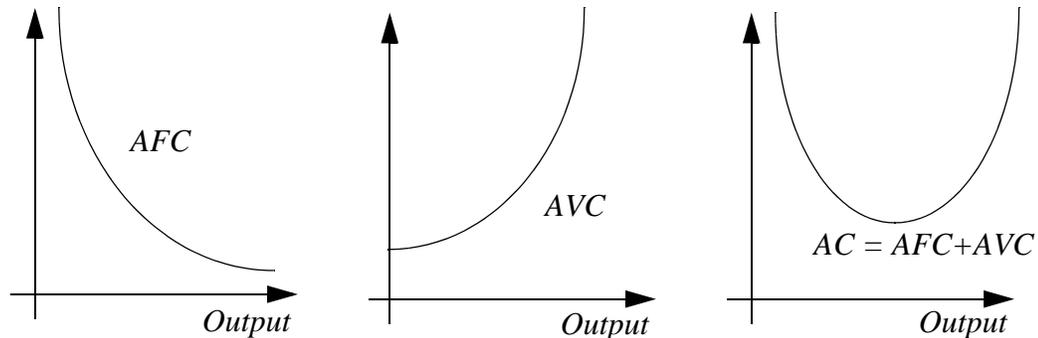
It is assumed that the fixed cost is incurred when the plant is on-line, meaning that the company can choose to take the unit off-line in which case the total cost would be zero.

Therefore, the total cost curve can be described as:

**Table 3-1: Total Cost**

Status	Output	Cost
ON	$P_i$	$Cost_i = F_i(P_i)$
ON	0	$Cost_i = F_i(0)$
OFF	0	0

The average cost function measures the cost per unit of output. This cost can be divided into two different components: Average Fixed Cost (AFC) and the Average Variable Cost (AVC). Examples of these curves are shown in the next figure.



**Figure 3-2: Variable Cost Functions**

The AFC is infinity at no-output level because of the fixed cost would be divided by zero, then it monotonically decreases. The AVC curve is always increasing.

The result of adding these two curves is the Average Cost, AC. From AFC and AVC it is known that AC starts very high when the output is close to zero influenced by the fixed cost. Then the curve decreases because the fixed cost is less important as the output increases and finally increases because of the influence of the AVC.

In describing the offer curve, it can be observed that the firm will never be willing to produce at a price less than the average cost. Moreover the company will always prefer to shutdown the plant before incurring losses for selling its energy at a price less than the average cost.

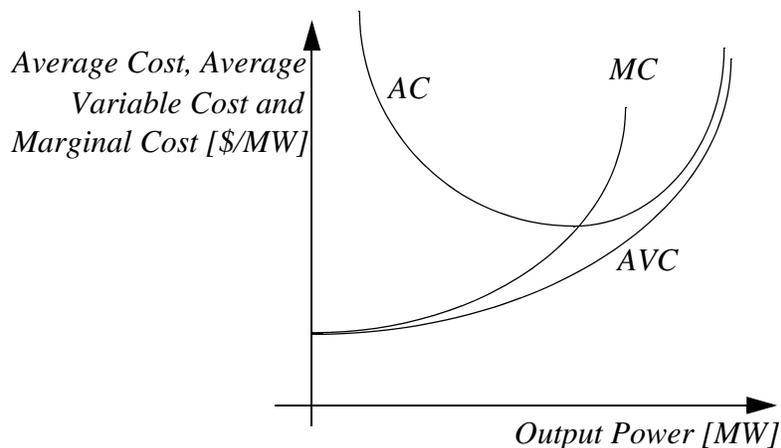
### 3.2.3 Marginal Cost

The Marginal Cost curve reflects the effect on the total cost for a given change in the

output level. From a mathematical point of view this curve is the first derivative of the total cost function, and therefore, does not depend on the fixed cost.

These two curves are related as follows:

- The Average Variable Cost and the Marginal Cost start from the same point (the average cost of producing one unit is the cost of the unit itself).
- If the Average Cost is decreasing then the Marginal Cost is less than Average Cost, and if the Average Cost is increasing the Marginal Cost is greater than the Average Cost.
- As a consequence of the last property, when Average Cost is neither increasing nor decreasing (i.e. at a minimum), both curves cross each other.



**Figure 3-3: Average Cost, Average Variable Cost and Marginal Cost vs. Output Power**

### 3.2.4 Economical Limit of the Offer

One implicit assumption of the analysis given in this section is that the generator company knows its own costs. This may sound trivial, but it is a complex and time consuming task inside a productive company. The assessment of its own costs gives a company an idea of where it is in the marketplace, and the minimum and optimum price it should be willing to offer to buyers.

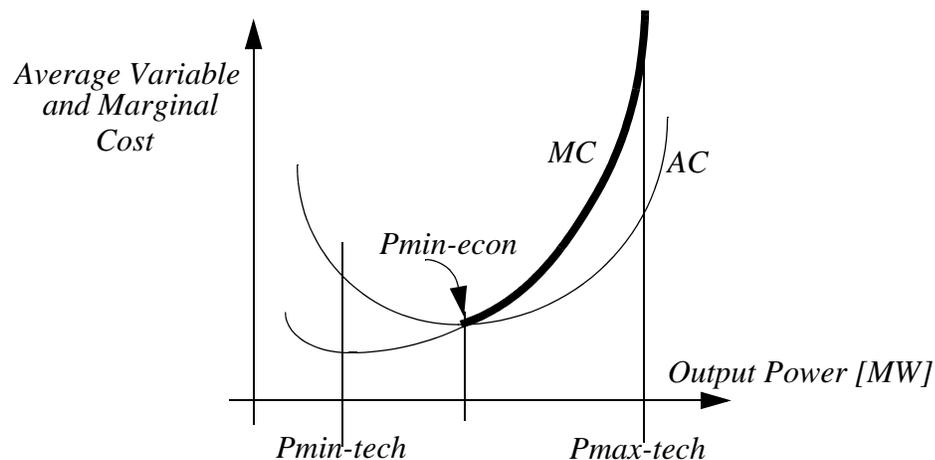
From an operations research point of view, given a certain price the optimum policy for a generator company is to operate where its marginal cost is equal to the price. This can be easily derived from the first order condition of the maximization of profits. An intuitive explanation of this can be found by examining two cases for an increasing Marginal Cost curve:

- **The Marginal Cost is greater than the Price Offered.** The generator company would be better off decreasing its production by one unit, because the revenue obtained from that unit is less than the cost to produce it.
- **The Marginal Cost is less than the Price Offered.** The generator company would be better off increasing its production by one unit, because the revenue obtained from that extra unit is more than the cost to produce it.

This is valid only when the Marginal Cost curve is increasing, giving the company a profit equal to or greater than zero. This means that the only case to consider is when

the Marginal Cost curve is greater than the Average Cost curve. (See subsection 3.2.3.)

Therefore combining the results of the last section with the above one can obtain the resulting offer curve. Summarizing, the domain of this curve will have a lower bound at the maximum of the minimum technical limit (minimum in which the generator can be operated) and the economical minimum (minimum of the average cost). The upper bound will be the technical limit (generator maximum output). The optimum offering for the generator company will always be its marginal cost under the domain defined above.



**Figure 3-4: Offer Curve**

### 3.3 Consumers and the Demand Curve

Extrapolation of the assumptions made in assessing the costs and benefits for electricity producers (generator companies) to the consumers (loads) can be

misleading.

In the case of profit-driven companies owning generators one can clearly state the costs and revenues. But in the case of consumers, one needs to be very careful in stating even the most basic rules.

It can be assumed that consumers have a set of preferences. These preferences follow certain rules in the case of electricity: strict monotonicity (more is always better) and continuity. So, independently of how much electricity a load is consuming, it will never hurt the load to have an extra kWh. Also, the marginal benefit for the load of the first kWh received is larger than the benefit of the second one and so on and so forth (decreasing marginal benefit).

Having these assumptions in mind, there are utility functions that can be used to represent the preferences of a consumer. Moreover, given a set of preferences for a certain consumer, there exists an infinite number of utility functions that can be used to represent such a preference.

As an example, the following two utility functions can represent the preferences of a consumer for a level  $P$  kWh of consumption.

$$U(P) = aP \text{ and } U(P) = b \ln P \quad (3.3.1)$$

Notice that any positive value for the scalar constants  $a$  and  $b$  can be used to represent

the consumer preferences (both functions would be monotonically increasing and continuous for any positive value of P).

Considering now the idea of efficiency (maximize the total utility), it is well known that the utilities of individual consumers cannot be added because the result of the optimization is going to depend on arbitrary utility functions that were used to represent the consumer's preferences.

There is also a basic problem in the optimization of multiple consumers when the result of surveys of the willingness of consumers to pay for electric power are used to obtain utility functions. The result of such an optimization can be economically optimum but not socially efficient. For example, it is known that the willingness to pay (in dollars) for the consumption of a first kWh of a wealthy consumer is greater than that of a poor consumer. Therefore, any optimization algorithm based on the maximization of just the sum of the individual benefits of all consumers is going to assign that first kWh to the wealthy consumer. The result may be economically optimum, but it is not likely to be socially efficient. Nevertheless, in the case of distribution companies and industrial consumers the idea of using *willingness to pay* (from internal research or surveying the company) is clearly the only (and best) alternative to represent them in a dispatch problem.

### **3.4 Individual Rationality**

The individual rationality property is based on the assumption that people and companies are rational and will always choose the alternative that maximize their benefit (profit), including the alternative of not participating. Therefore, if any consumer or generator company chooses a non-zero alternative, this alternative should provide it a non-negative profit, otherwise it would rather not produce and get zero profit.

In this application it was assumed that every consumer or generator company has the alternative of not participating in the market. For a generator company this means the possibility of turning off the generator, zero revenue and zero cost, resulting in a null profit.

### **3.5 Incentive Compatibility**

The incentive compatible property means that the optimal action for each participant is to quote its true private information (true marginal cost or true willingness to pay) provided that all other participants quote their true information also. To achieve the incentive compatible property in the electricity market, sufficient incentives must be provided to both consumer and generator companies, so that action which maximizes profits for each results in the submission of true marginal costs or willingness to pay. Notice that incentive compatibility does not mean that a participant would always tell the truth; instead, it means “I would tell the truth if others also tell the truth.”

### 3.6 Markets and Auctions

A market is a place where consumers and producers are brought together to interchange goods (with money also treated as a good). A market can be set up with or without a third party that coordinates the interactions among participants. Two different market schemes can be proposed:

- The first is a bilateral trading market. In this scheme, two cases arise. In the first the seller places a higher value on the product than the buyer (because it costs more to produce than the benefit the product brings to the buyer). There is no gain to such a transaction, and therefore there is no market activity. In the second case, the seller places a lower value on the product than the buyer, and therefore, there exists a net gain from such trading. Which participant receives the surplus depends on the bargaining power of the participants and/or the set of rules fixed by the market structure.
- The second market scheme is an auction. Here a central authority (auctioneer) asks the participants (bidders) to provide a quote reflecting the value of a product to them. In a classical auction the central authority may be the seller (just one) and all the participants, the buyers. After all the participants have submitted bids for the product the central authority assigns both the product and a payment for it, following a preestablished set of rules. There is extensive literature covering the several auction schemes that are currently in used. (Mas-

Colell includes such a description, [Mas-C95].)

Either of these schemes or both simultaneously may be used in the electric power marketplace. A common mix of both schemes will run a bilateral market for the long and mid-term transactions and an auction to smooth out the short-term differences between offer and demand (spot market).

### **3.7 Monopoly and Market Power**

The success of competitive markets is based on two fundamental aspects: first, under certain conditions they are efficient (Pareto efficient), second, they are easy to establish. Why is a competitive market efficient? Economists have formalized a theorem (the First Welfare Theorem) that demonstrates that given certain conditions (such as a finite number of participants) a competitive equilibrium leads to efficiency (in a Pareto sense), [Mas-C95]. It is known that some of these conditions hold in the case of the electricity market (such as finite number of participants), but it is also known that there is particular difficulty in obtaining the price-taking behavior. This behavior comes from the notion that producers and consumers perceive that they can not modify prices. This is the case of a competitive market with a very large number of competitors, in which these competitors take prices as given, but is not true in a monopoly in which the producer fixes the prices according to its own criteria. Most of the cases are in between these two extremes.

In the case of the electricity marketplace, the price-taking behavior assumption may not be reasonable because most of the time there are only a few (or just one) agents on either side. The agent having influence over the price is said to have *market power* - the ability to alter

prices away from competitive levels so as to gain greater profits (details can be found in [Mas-C95]).

Market power makes efficiency less likely to be achieved in a monopoly setting. In a monopoly there is one producer and this producer will realize the effect of its offered price, and therefore will manipulate the prices in order to increase its profits. The monopolist will notice that an increase in price only carries a small reduction in its sales, finding an equilibrium (in which its profit is maximized) results in prices different from those in a competitive market. The same distortion may occur when no monopoly exists, but the number of competing producers is small.

In the case of the electric power marketplace there are two main reasons why some agents may have market power:

- A generator company whose units are critical (the remaining part of the system cannot serve the load without them) has a competitive advantage with respect to the rest of the market. This means that this company can raise its price knowing that some of its power it is going to be bought anyway. For example, having a one-bus system with a fixed load of 100 MW and two generator companies, the first one owning a unit of 110 MW capacity and the second owning a 50 MW capacity unit. The first company knows its power is critical to serve the load. Moreover, it knows that it can raise its price as much as it wants and the load must pay what is asked if it wishes to maintain its supply. The second generator company, on the other hand, knows that if it raises its price it will sell no power. The first company is said to have market power.

- The second source of market power appears when a transmission network limit is superimposed on the first situation. The importance of a generator company is increased by the fact that the limited capacity of the transmission network helps to isolate it from the remaining generators.

The second condition above is frequently found in electric power markets, even when they are large. Notice that a generating company that has some market power stops behaving as a competitive participant and starts behaving as a monopolist. Therefore the benefits of a competitive market are partially or completely lost.

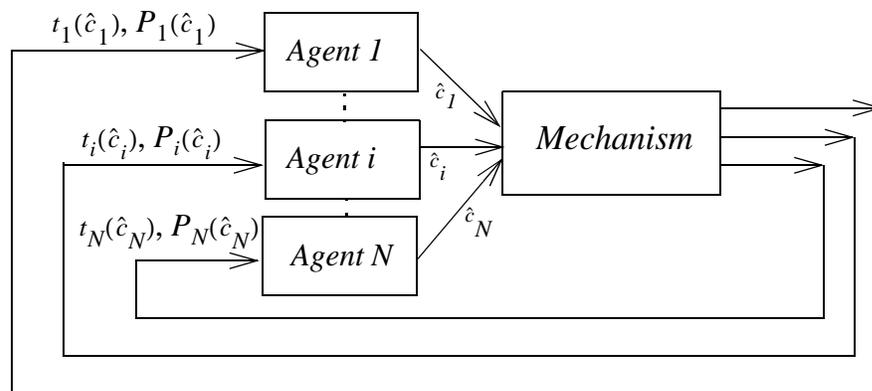
### **3.8 Mechanism Design**

Economists use the word mechanism to describe a set of rules defining a game played by a group of participants. The purpose of these rules is to achieve a certain outcome by providing appropriate incentives to the participants. A mechanism may be decentralized (free markets, open auctions, etc.), centralized (the military, etc.), or any form in between (government-regulated markets, etc.).

With respect to power system markets, there is a focus on the following type of mechanisms: Each participating generator submits a marginal cost and each participating consumer submits a willingness to pay (neither the submitted marginal cost nor the submitted willingness to pay need to be the true values for the participants) to an entity (coordination entity, pool operator, power exchange, etc.). Given the

submitted values, the agent allocates production and payments to the participants. Notice that any participant sees the agent's allocation and payment as optimum since it knows its own cost or willingness to pay and uses the expected values for the remaining agent's cost or willingness to pay. (This is the concept of *interim* optimum [Mas-C95].)

This is illustrated by Figure 3-5, where  $\hat{c}_i$  is the marginal cost or willingness to pay submitted by  $i$  agent,  $t_i$  is the payment to agent  $i$ , and  $P_i$  is the production or consumption assigned to agent  $i$ .



**Figure 3-5: Mechanism**

In general, a mechanism is designed so as to satisfy several properties in order for it to serve the designer's objective. These properties may include: incentive compatibility, efficiency, individual rationality, etc.

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## CHAPTER 4 ELECTRIC ENERGY MARKET ORGANIZATION

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An electric power market designer (normally a regulator) has several alternatives to structure and operate the market. These alternatives can be described using the payment scheme that is applied to the market participants.

A payment scheme is defined as the compensation the generator companies get and load companies pay for the electric energy they provide or receive.

The efficiency of any market depends on the payment scheme used. In this section some of the currently proposed payment schemes are reviewed.

### **4.1 The Traditional Payment Scheme**

In this scheme, participants (generators and consumers) are paid the declared price (generators) or pay their declared bid to purchase (consumers). This scheme can be understood as *naive* because the coordination entity (power pool) simply acts on the participants declarations directly.

For a generator company at bus  $i$ , involved in a transaction to a load at  $j$ , is selling

$P_{tran_{ij}}$  MW, the cost will be,

$$Cost = TrueCost \cdot P_{tran_{ij}} \quad (4.1.1)$$

and the revenue,

$$Revenue = QuoteToSell \cdot P_{tran_{ij}} \quad (4.1.2)$$

and finally the profit,

$$Profit = (QuoteToSell - TrueCost) \cdot P_{tran_{ij}} \quad (4.1.3)$$

For the load company buying  $P_{tran_{ij}}$  MW, the cost will be,

$$Cost = QuoteToBuy \cdot P_{tran_{ij}} \quad (4.1.4)$$

and the revenue is the amount of power times consumer company's true willingness to pay (the willingness to pay corresponds to a dollar amount of the utility the company gets by consuming the energy, for more details see subsection 3.3),

$$Revenue = WillingnessToPay \cdot P_{tran_{ij}} \quad (4.1.5)$$

and finally the profit,

$$Profit = (WillingnessToPay - QuoteToBuy) \cdot P_{tran_{ij}} \quad (4.1.6)$$

## 4.2 Bus Marginal Cost Payment Scheme

In this scheme generator companies are paid and consumer companies pay the marginal cost at the bus where they are located.

For a generator company at bus  $i$  selling  $P_{tran_{ij}}$  MW, the cost will be,

$$Cost = TrueCost \cdot P_{tran_{ij}} \quad (4.2.7)$$

and the revenue,

$$Revenue = MCost_i \cdot P_{tran_{ij}} \quad (4.2.8)$$

and finally the profit,

$$Profit = (MCost_i - TrueCost) \cdot P_{tran_{ij}} \quad (4.2.9)$$

For a load company at bus  $j$  buying  $P_{tran_{ij}}$  MW, the cost will be,

$$Cost = MCost_j \cdot P_{tran_{ij}} \quad (4.2.10)$$

and the revenue,

$$Revenue = WillingnessToPay \cdot P_{tran_{ij}} \quad (4.2.11)$$

and finally the profit,

$$Profit = (WillingnessToPay - MCost_j) \cdot P_{tran_{ij}} \quad (4.2.12)$$

Notice that if a load company located at bus  $j$  is buying from a generator at bus  $i$ , the company is paying the cost of generating at  $i$  ( $MCost_i$ ) plus the cost of losses and congestion to transport the energy to bus  $j$  ( $MCost_j - MCost_i$ ). This payment scheme is compatible with a market because the participants do not directly fix their own payments and the network is partially considered through the use of marginal costs. Nevertheless, in the simulation sections it will be later shown that this mechanism is not necessarily sufficient to establish a competitive and fair marketplace.

### 4.3 The Transmission Congestion Contract Payment Scheme

This section describes a methodology using Transmission Congestion Contracts. This new approach was first proposed by William W. Hogan from Harvard University [Hogan92] and is presently being used by some system operators. This section briefly explains the idea behind this methodology and gives some discussion of its applicability in electric power markets.

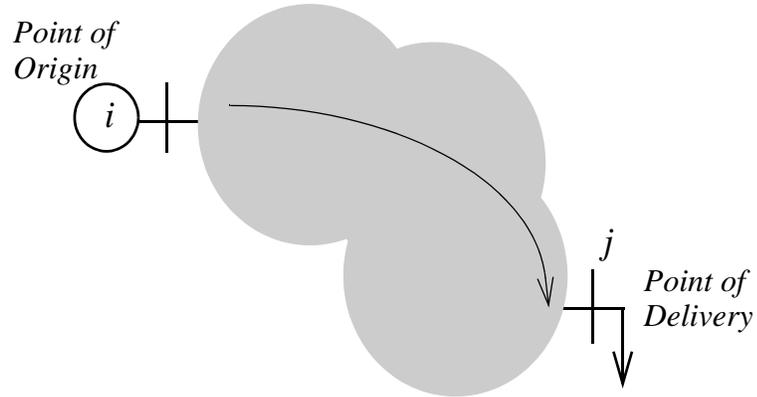
A Transmission Congestion Contract was proposed as a method of dealing with some of the deregulation issues and achieving short-term efficiency in the market. Hogan introduced the idea of an asset (a congestion contract) that will eliminate the risk of not

being able to fulfill a contract between a generator and a load due to inadequate transmission capacity.

Transmission Congestion Contracts are to be traded in a secondary market (a different market from the main electric power market). A Transmission Congestion Contract gives the owner the right to transfer power from bus  $i$  to  $j$  without paying a congestion fee (difference in marginal cost due to congestion). The main advantage of not paying for congestion is that agent does not know in advance how high the congestion charge might go, so it is preferable to be independent of this risk.

The Transmission Congestion Contract can be set up as follows: participants (generators and consumers) are paid or pay the marginal cost at each bus. Companies owning congestion contracts receive a payment equal to their power flowing through the transmission network from the point of origin,  $i$ , to the point of delivery,  $j$ , times the difference in the marginal costs from  $i$  to  $j$  (the congestion charge).

A basic example is shown in Figure 4-1, having a two-bus contract in a large system. (For the sake of simplicity losses are neglected.)



**Figure 4-1: Transfer between Generator i and Load j**

Assume that generator company i owns a Transmission Congestion Contract from i to j for a certain power  $P_{tran_{ij}}$ . If there is no congestion, all buses are going to have the same marginal cost, and generator i is going to be paid  $MCost_i = MCost_j$  for the each MW. Therefore its cost and revenue will be:

$$Cost = TrueCost \cdot P_{tran_{ij}} \quad (4.3.1)$$

$$Revenue = MCost_i \cdot P_{tran_{ij}} \quad (4.3.2)$$

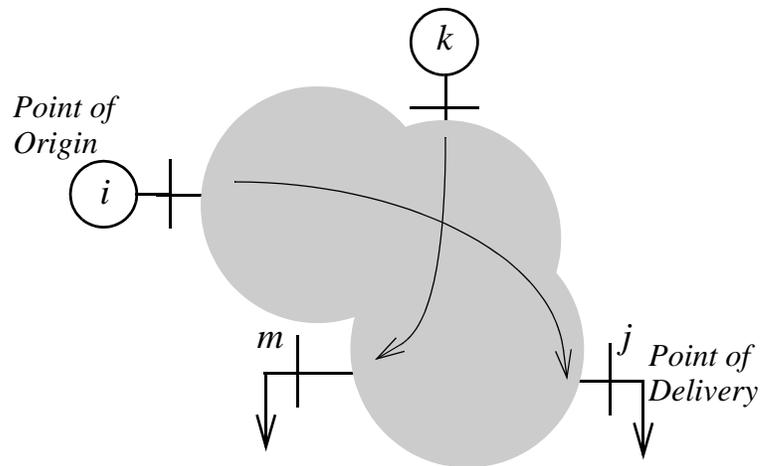
and finally the profit,

$$Profit = (MCost_i - TrueCost) \cdot P_{tran_{ij}} \quad (4.3.3)$$

This is exactly the same profit the company would receive using the bus marginal cost

payment scheme, eq. (4.2.9).

If a new transfer is introduced between generator  $k$  and the load  $m$ , causing congestion in the path  $ij$ , the price at  $j$  is going to increase from  $M\text{Cost}_{j0}$  to  $M\text{Cost}_j$  (greater than  $M\text{Cost}_i$ ).



**Figure 4-2: Generator  $i$  and Load  $j$  with Additional Transfer**

In the case of not owning a congestion contract, this would force generator  $i$  to pay a transmission losses and congestion charge equal to the its transfer times the difference in bus marginal cost between buses  $i$  and  $j$ . But since the company owns a contract it

does not need to pay the congestion charge. Therefore, its cost and revenue will be:

$$Cost = TrueCost \cdot P_{tran_{ij}} \quad (4.3.1)$$

$$Revenue = (BusMarginalCost + (Congestion + Losses)) \cdot P_{tran_{ij}} \quad (4.3.2)$$

$$Revenue = MCost_i \cdot P_{tran_{ij}} + (MCost_j - MCost_i) \cdot P_{tran_{ij}} = MCost_j \cdot P_{tran_{ij}} \quad (4.3.3)$$

and finally the profit,

$$Profit = (MCost_j - TrueCost) \cdot P_{tran_{ij}} \quad (4.3.4)$$

Notice that generator i is indifferent to the following two alternatives:

- Generate power at bus i with a cost of  $MCost_i \cdot P_{tran_{ij}}$ , pay for congestion and losses  $(C_j - C_i) \cdot P_{tran_{ij}}$ , and finally get the congestion payment from its contract of  $(C_j - C_i) \cdot P_{tran_{ij}}$ .
- Buy the power at j at  $MCost_j \cdot P_{tran_{ij}}$  and use the income coming from its contract,  $(C_j - C_i) \cdot P_{tran_{ij}}$ , to subsidize its cost.

The difference between the strict Bus Marginal Cost scheme and the Transmission Congestion Contract scheme is that a participant buying a contract for a transfer between buses i and j makes a profit which depends on the congestion on this path. Notice that in order to analyze the cash flows of the Transmission Congestion Contract

payment scheme one needs to assume a price for the contract itself.

#### 4.4 Incentive Compatible Payment Scheme

In this scheme generators are paid their offered price and consumers pay their declared willingness to pay. In addition they are paid an added amount called the information incentive payment. This incentive payment has the purpose of eliminating any incentive among participants to change their quotes from the true cost or true willingness to pay. (More details on this mechanism are found in Chapter 5.)

For a generator company selling  $P_{tran_{ij}}$  MW, the cost will be,

$$Cost = TrueCost \cdot P_{tran_{ij}} \quad (4.4.5)$$

and the revenue,

$$Revenue = QuoteToSell \cdot P_{tran_{ij}} + InfoPayment(QuoteToSell) \quad (4.4.6)$$

and finally the profit,

$$Profit = (QuoteToSell - TrueCost) \cdot P_{tran_{ij}} + InfoPayment(QuoteToSell) \quad (4.4.7)$$

For a load company buying  $P_{tran_{ij}}$  MW, the cost will be,

$$Cost = QuoteToBuy \cdot P_{tran_{ij}} \quad (4.4.8)$$

and the revenue,

$$Revenue = WillingnessToPay \cdot P_{tran_{ij}} + InfoPayment(QuoteToBuy) \quad (4.4.9)$$

and finally the profit,

$$Profit = (WillingnessToPay - QuoteToBuy) \cdot P_{tran_{ij}} + InfoPayment(QuoteToBuy) \quad (4.4.10)$$

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## CHAPTER 5 THE INCENTIVE COMPATIBLE MECHANISM

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### 5.1 Introduction

This chapter presents the incentive compatible mechanism, a new methodology to deal with congestion management and the creation of a competitive market in power systems. The main difference between the methodologies described in Chapter 4 and the incentive compatible mechanism is the assumption that when participants quote their willingness to buy or sell power, they choose the quote that maximizes their profit and not necessarily its true cost or true willingness to pay. Although this assumption may seem reasonable, most of the current methodologies do not recognize its importance.

### 5.2 Information Assumption Background

Every company in the electric power market knows information about itself that every other company ignores (supply contracts, efficiency of the machinery, etc.). In fact, other companies or a central authority may have an idea of what its costs are, but never with certainty.

Companies will use this *private* information (cost for generators and willingness to pay for consumer companies) to their own benefit. From a mathematical point of view, a company is going to maximize its own profit function. Also, the fact that this company

is hiding its true cost function gives it another variable to play in the market game. Moreover, no rational company will reveal its private information, because this would give an advantage to its competitors.

This behavior is already being seen in markets, but it is also clear that the transition from a regulated to a deregulated environment has some intermediate states. If, as a starting point, a group of utilities share critical information, and then are required to work separately, it would be predictable to have them still cooperate for some time. But in the mid-term, they are going to figure out that they can not give up any competitive advantage by giving key information to their competitors. Starting from that point, these utilities will begin behaving more aggressively, and the benefits of open markets are going to be perceived.

### **5.3 The Mechanism**

As discussed in subsection 3.8, a mechanism assigns production and payments to participants in the market as functions of the marginal costs and willingness to pay. To assure that the mechanism is feasible and achieves efficiency, each production assignment needs to be a solution of the economic dispatch problem based on the *true* marginal costs and true willingness to pay. Consequently, if it is temporarily assumed that participants tell the truth due to a suitable payment function (specified later), the assignment function is easy to derive: it is a solution of the economic dispatch problem based on the submitted generator company marginal costs and consumer company

willingness to pay.

What is needed is a payment scheme that makes the mechanism incentive compatible, i.e., that induces each participant to submit its true marginal cost (generators) or true willingness to pay (consumers). To do this, the operator looks at each participant's optimization of what to submit. If a mechanism is indeed incentive compatible, then each participant would think it is optimal to submit its true marginal cost or true willingness to pay. By solving this optimization problem, the possible candidates for such a payment scheme are reduced to a family of payment functions that differ from one another by a constant. One then chooses the payment scheme that satisfies the individual rationality condition. (For details of how to derive this function see Appendix A).

#### **5.4 Description of the Incentive Compatible Mechanism**

The following mechanism was first proposed and proven correct for a 3-bus network in [Zheng99], and Chapters 7 and 8 show the results of simulations indicating that the mechanism has all the desirable properties.

The general implementation rules are the same for all the simulations included in this thesis. In the case of the incentive compatible mechanism, the procedure goes as follows:

- Ask participant  $i$  (from  $i=1$  to the number of participants) to submit its marginal cost or willingness to pay,  $\hat{c}_i$ .
- Assign to participant  $i$  the production resulting from the economic dispatch as described in subsection 2.1 which uses the marginal costs and willingness to pay the participants have submitted. Let  $P_i(\hat{c}_1, \dots, \hat{c}_{N_g})$  denote the quantity participant  $i$  is assigned to produce or consume (depending on the sign).

Pay each generator company and collect from each load company a “cost compensation”,

$$\hat{c}_i \cdot P_i(\hat{c}_1, \dots, \hat{c}_{N_g}) \quad (5.4.1)$$

and an “information compensation” (for details see Appendix A),

$$\tau_i(\hat{c}_1, \dots, \hat{c}_{N_g}) = \frac{1}{\Psi_i(\hat{c}_i)} \int_{\hat{c}_i}^{\bar{c}_i} \bar{P}_i(x) dx \quad (5.4.2)$$

where  $\hat{c}_i$  is the marginal cost or willingness to pay provided by  $i$  (in the form of a quote to buy or sell),  $\bar{c}_i$  is the maximum possible marginal cost or willingness to pay for  $i$ ,  $\bar{P}_i(\hat{c}_i)$  is the expected production for generator  $i$  (or consumption for load  $i$ ). The term  $\Psi_i(\hat{c}_i)$  is the probability assessed by generator or load  $i$  to the event that it will be assigned to produce or consume something (different from zero).

The above mechanism achieves all of the following:

- It induces every participant to provide its true marginal cost or willingness to pay (incentive compatibility).
- It assigns production efficiently (feasibility and efficiency).
- It guarantees that no generator company or load company loses profit if it submits its true marginal cost or true willingness to pay (individual rationality).

### **5.5 Interpretation of the Payment Scheme**

The payment consists of two parts. The first is a cost compensation according to the marginal cost quoted by the generators and willingness to pay quoted by the loads. The second, which is indispensable due to the information structure, is a compensation to the participant for providing true cost or true willingness to pay. This part is precisely tuned so that each participants is willing to provide the true marginal cost (generator) and true willingness to pay (load).

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## CHAPTER 6 SIMULATION TECHNIQUES

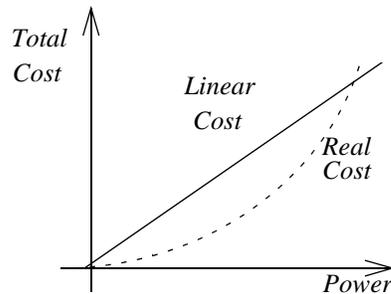
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This chapter explains the simulation technique used to contrast the variations of the bus marginal cost scheme from Chapter 4 and the incentive compatible mechanism described in Chapter 5. The following is the set of assumptions used in the simulation. The results from these simulations are seen in Chapter 7 for a single period agent optimization, and in Chapter 8 for a multi-period form.

### 6.1 Physical Environment Assumptions

This section describes the assumptions that are related to the network itself and the technical aspects of the agents.

- **Linear Cost Functions:** In order to simplify the application of mechanism design, this thesis assumes that a generator company cost and a load company willingness to pay is a linear function of its production (generator) or consumption (load). Thus, a cost or willingness to pay is transformed to a straight line. In the case of a generator its real cost and the linear representation of it would appear as in Figure 6-1:



**Figure 6-1: Real and linear total cost**

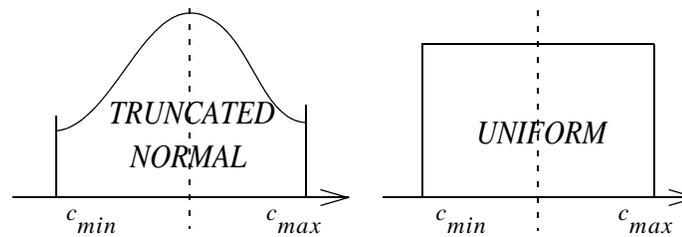
- **Load.** This thesis assumes that loads can be either constant or variable. For the constant load the demand for electric power at bus  $i$  is a given value, and for the variable load, the load company is included in the simulation as an agent having limits on what it can consume and a private willingness to pay.
- **Non-Empty Feasible Set.** It is assumed that the feasible set defined by the DC power flow constraints, the limits of the generators and loads and the limits on transmission line flows is non-empty. Without such an assumption, the economic dispatch problem would have no solution.

## 6.2 Information Problem Assumptions

This subsection formulates the above into the following assumptions.

- **Private and Public Information.** The true marginal cost of a generator is assumed to be private information, meaning that a generator knows its own inputs (e.g., fuel, boiler efficiency, etc.) better than anyone else. With generators competing in a deregulated environment, they are not willing to quote their true cost unless given sufficient incentive to do so. One can also assume that a generator  $j$  regards the marginal cost of another generator  $i$  ( $i \neq j$ ) as a random variable from a commonly known distribution. This common knowledge may be obtained from the publicly available information about the other generator's technology, prevailing fuel prices, etc. (and any other information competitors can obtain just looking at the plant). The other parameters of an electric system, including transmission line capacities, line admittances, fixed loads and the technical limits of the generators, are assumed to be common knowledge among all market participants.

- **The Distribution of Marginal Costs:** it is further assumed that the distribution of a generator's (true) marginal cost has the following properties:
  - (i) The domain of possible marginal cost is bounded. In other words, others know that generator  $i$ 's marginal cost lies between ceiling and floor values.
  - (ii) The probability density function of the distribution is continuous. Many distribution functions satisfy the above properties. Figure 6-2 shows two of such examples.



**Figure 6-2: Distribution Functions of the True Marginal Cost**

### 6.3 Exhaustive Enumeration

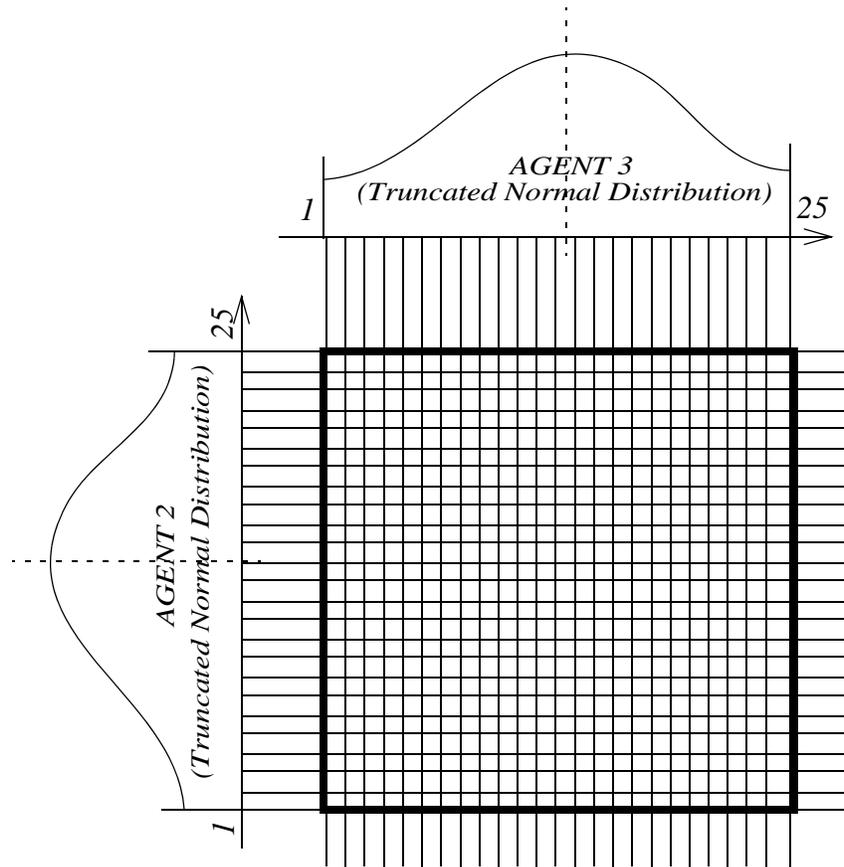
The information assumption given in subsection 6.2 describes the perception of any agent about the marginal cost of a generator (or willingness to pay of a load) as a probability distribution. Therefore agent  $i$  believes that the quotes of the other agents are going to be drawn from that specific probability distribution. Based on this assumption, any agent can calculate its own expected profit, assigned production or any other parameter by simply using the probability distributions of the remaining participants.

There are two different ways in which an agent can calculate an expected value: exhaustive enumeration or Monte Carlo simulation. The former is described in this subsection and the latter in the next.

The idea of an exhaustive enumeration is to first approximate the probability distribution of a generator's marginal cost or load's willingness to pay by a finite grid, defining a set of possible costs, each one with an associated probability. For each combination of marginal costs for generators and loads (for all agents other than  $i$ ),

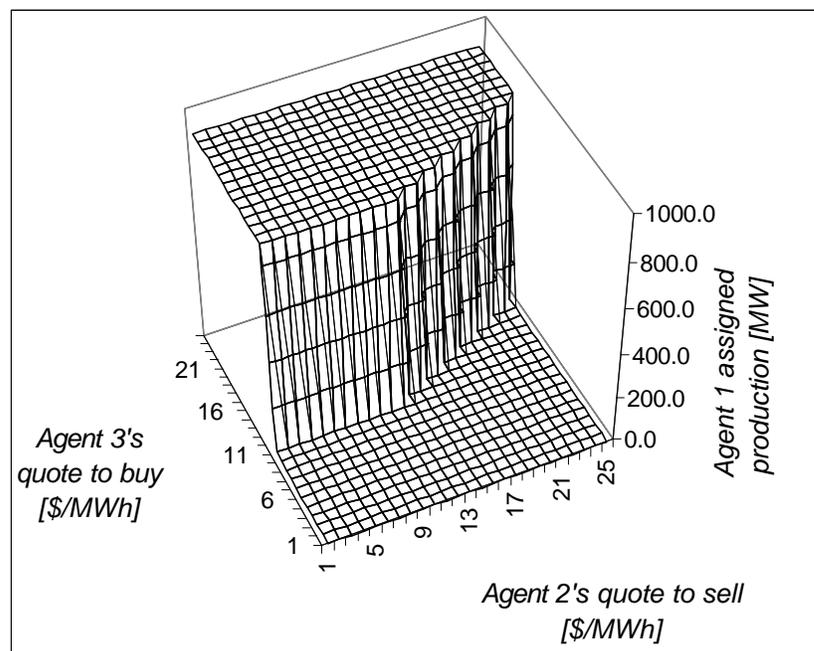
agent  $i$  can calculate the corresponding expected value.

As an example, in a 3-bus case with 3 agents (data is in Appendix C) the generator company located at bus 1 (agent 1) would like to know its expected assigned production in the case of quoting to sell at 12 [\$/MWh]. The first step is to apply a grid to the distribution function of each remaining participant. The grid for agent 2 and agent 3 is shown in the following figure.



**Figure 6-3: Grid to Calculate Expected Values for Agent 1**

Every point in this grid has an associated vector of quotes (one for agent 2 and one for agent 3) together with an associated vector of probabilities. Using these vectors and its own quote, agent 1 can use an Optimal Power Flow to calculate its production assignment for each point shown in Figure 6-3. The resulting grid is shown in the following figure.



**Figure 6-4: Power Assigned to Agent 1 (Generator) for the Different Quotes from the Remaining Agents (assuming agent 1 used 12 [\$/MWh])**

Agent 1 will then weigh this result using the probability assigned to each point of Figure 6-3. The result will correspond to the expected production assignment for this agent if it used 12 [\$/MWh] as its quote to sell.

Normally, agent 1 will obtain the quote value that maximizes its expected profit by repeating the above process for all possible quote to sell values, and a given payment scheme (payment schemes are described in Chapter 4 and Chapter 5), and then simply selecting the optimum from the set of outcomes.

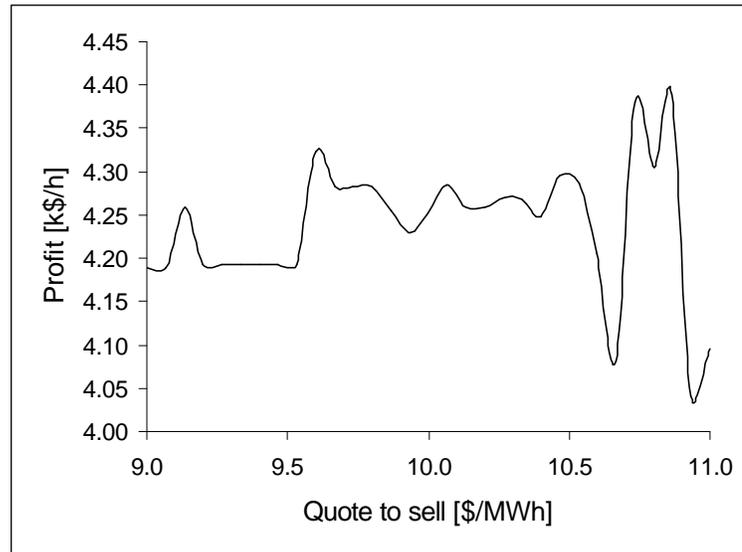
## 6.4 Monte Carlo Simulation

The term Monte Carlo simulation is often used to refer to a simulation in general. Technically (from [Hilli95]) the Monte Carlo technique is referred to as a variance-reducing methodology. The idea is to generate an estimate of the relevant output of a system using a group of random inputs.

The key point is a variance-reduction, the more individual simulations performed the better the approximation of the outputs. This gives the researcher a trade-off between accuracy and processing time for the simulations.

An example of the importance of processing time is given in the following application of a 3-bus system. In this system every agent (either a generator or a load) assumes that the remaining agent's cost or willingness to pay varies according to a certain probability distribution. The agent knows that the more time it dedicates to simulating the system the better the approximation of an optimal policy. Figure 6-5 shows the expected profit for agent 1 (a generator) for different quotes considering 10 simulations

per quote (30 levels).

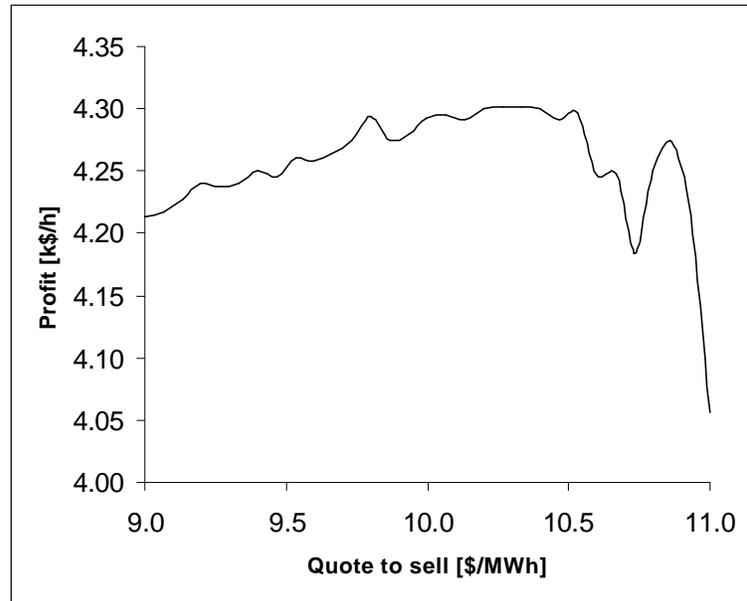


**Figure 6-5: Profit for Agent 1 Using 10 Simulations per Quote**

Using Figure 6-5, agent 1 would wrongly conclude that it is in its best interest to quote 10.733 [\$/MWh] (as is going to be seen later, the quote that gives agent 1 the maximum expected profit is 10.333 [\$/MWh]). From the conditions of the problem, agent 1 may know that the profit curve is continuous and has just one maximum, therefore it can safely conclude that the policy obtained from considering just 10 simulations per quote is not enough to decide on an optimal quote.

Following this rational agent 1 decides to decrease the variance in its curve by increasing the number of simulations. Considering 100 simulations (10 times more) per

quote the expected profit vs. quote to sell can be seen in Figure 6-6.

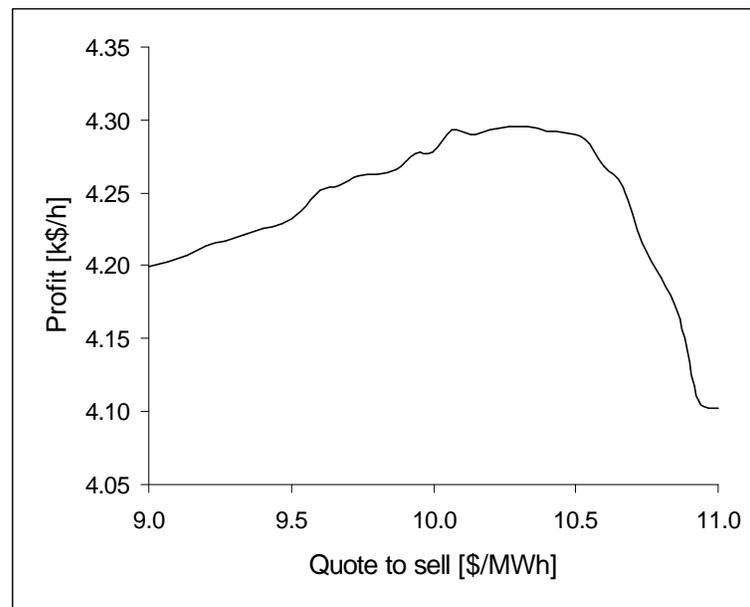


**Figure 6-6: Profit for Agent 1 Using 100 Simulations per Quote**

The process to obtain this curve takes 10 times more processing than the previous one, but the curve gives agent 1 a much clearer idea of what the profit curve looks like. None the less, from this curve agent 1 would wrongly assume that it is in its best interest to quote 10.267 [\$/MWh] (the quote that gives agent 1 the maximum expected profit is 10.333 [\$/MWh]). Again, agent 1, knowing that the profit curve is continuous and has just one maximum, can safely conclude that the policy obtained from considering 100 simulations per quote is still not enough to decide in an optimal claim.

Following this rational, agent 1 decides to decrease the variance in its curve even more,

this time augmenting the number of simulations to 1000 (10 times more). For this amount of simulation the expected profit vs. quote to sell appears as in Figure 6-7:



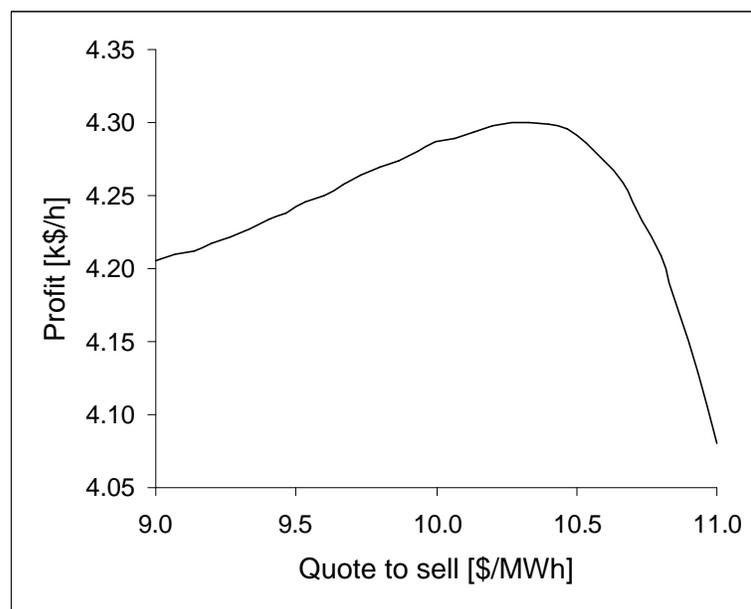
**Figure 6-7: Profit for Agent 1 Using 1000 Simulations per Quote**

Using this new curve, it is much easier for agent 1 to decide on a quote that will maximize its expected profit. Again, the process to obtain this curve is 10 times more time consuming than the previous one, but the curve gives agent 1 a much clearer idea of what the profit curve looks like. But even having a more defined curve agent 1 would wrongly assume that it is for its best interest to quote 10.267 [\$/MWh] (the quote that gives agent 1 the maximum expected profit is 10.333 [\$/MWh]).

This is very common in Monte Carlo simulations, even using a large number of

simulations, the result still is not exact and there is still the possibility of error. Nevertheless, in the case of continuous functions the mismatch between the true optimal value and the expected value decreases as the number of simulations increases.

Finally, agent 1 may decide to estimate the profit curve using 10000 simulations per quote. The resulting curve appears as in Figure 6-8:



**Figure 6-8: Profit for Agent 1 Using 10000 Simulations per Quote**

Using this new curve it is much easier for agent 1 to decide on a quote that will maximize its expected profit. Again, the process to obtain this curve is 10 times more time consuming than the previous one, but the curve gives agent 1 a much clearer idea of what the profit curve looks like. Using this curve agent 1 would correctly assume

that it is in its best interest to quote 10.333 [\$/MWh], which is the quote that maximizes its expected profit.

The following table shows the trade-off between accuracy and processing time. (This results approximated, in order to get a more precise curve a statistical analysis would be required.)

**Table 6-1: Example of Trade-off between Processing Time and Error**

Number of Simulations per quote	Processing time [s]	Quote that maximizes the expected profit [\$/MWh]	Error from real solution [\$/MWh]
10	7.4	10.7333	0.4
100	83.4	10.2667	0.0666
1000	770.0	10.2667	0.0666
10000	8908.1	10.3333	0

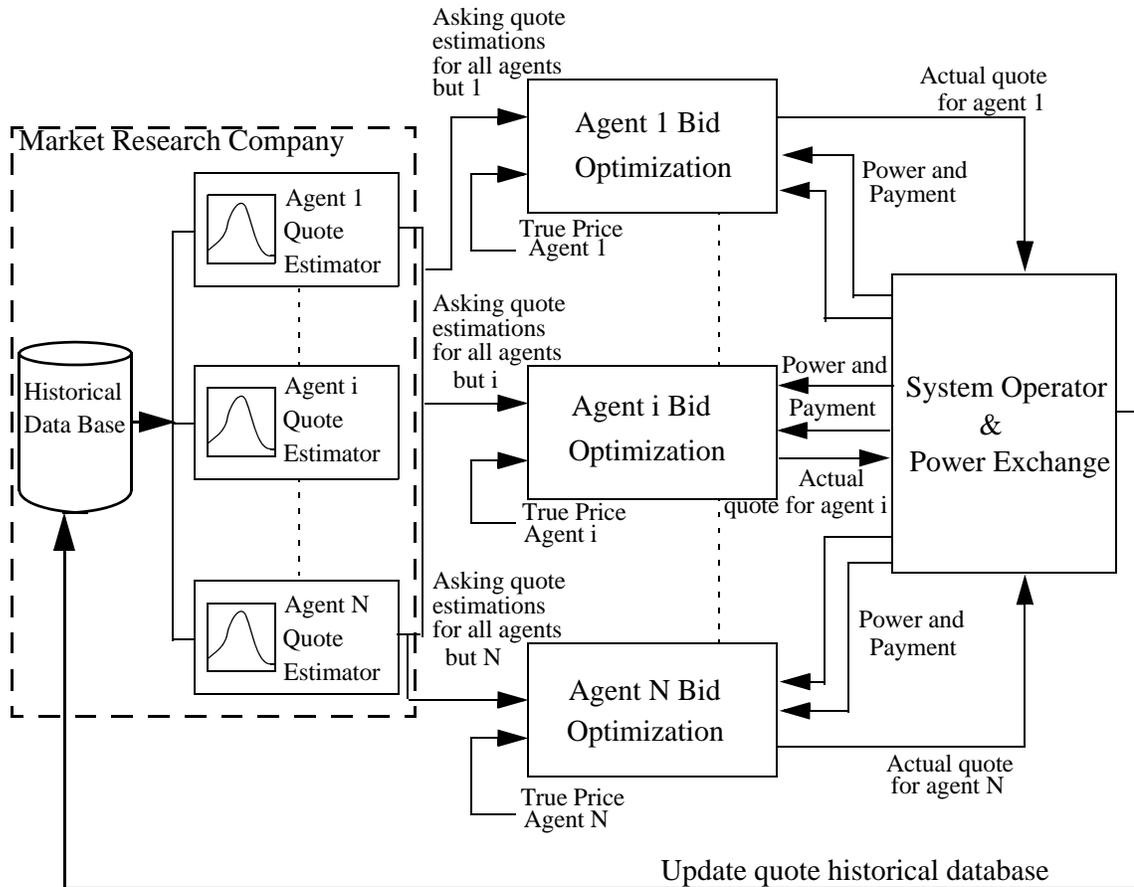
Summarizing, a Monte Carlo simulation has the advantage of being very fast if an accurate result is not important. Moreover the most important advantage is that researcher can choose the level of accuracy for a given available processing time. The disadvantages of this technique are: it takes a long time if an accurate solution is required; the estimates are not smooth and it does not consider extreme values (cases that are not likely to occur).

## 6.5 Multi-period Algorithm

The multi-period algorithm uses the same process described previously (using exhaustive enumeration, Monte Carlo simulation, or a combination of both). The additional information required is how to couple the periods among them (in this case period  $n$  uses the results from period  $n-1$ ).

The multi-period algorithm assumes the presence of a *Market Research Company* which provides information (about the distribution of marginal costs and willingness to pay) to the participants (generators and consumer companies) about the system.

The process starts with every agent acquiring the available marginal cost and willingness to pay information on its competitors from the Market Research Company. Then after some time processing the information, each agent has to decide what to quote (asking cost for the generators or a willingness to pay for loads). These values are passed from every agent to the System Operator & Power Exchange. This system operator runs an optimal power flow (using the quotes given) and calculates a payment to the agents (the payment scheme may vary). This allocation of production or consumption and the associated payments is then fed back to every agent and to the historical data base used by the Market Research Company.



**Figure 6-9: Overall Procedure for a Multi-period Simulation**

### 6.5.1 Market Research Company

This company stores and updates an estimate of the cost or willingness to pay for every agent in the system. These estimates are based on a probability distribution. This probability distribution itself is updated every period given the new information available from the previous period. In the technical literature there are several alternative algorithms to update the probability distribution. In the application shown

in Chapter 8 the Market Research Company uses a very simple version consisting of a list of numbers. Whenever a new estimate is available that estimate is appended to list. The user needs to calibrate the model for a given mean and variance and how fast these parameters change when new estimates are available.

### **6.5.2 System Operator & Power Exchange**

This entity receives all the cost/willingness-to-pay values from all the agents participating in the system. Using this information, it runs an optimal power flow to obtain the production and consumption assignment for every agent. It also calculates the payment that is going to be given to every agent.

### **6.5.3 Agent i Algorithm**

Agent i starts its algorithm by requesting the price estimates for all the other participants in the market (from the Market Research Company). At the same time agent i requires its own company to obtain its true cost curve (or if a consumer, its own willingness to pay). The agent now has an estimate for the remaining agents in the form of a probability distribution for each plus a realistic curve describing its own characteristic.

Agent i can also replicate the System Operator & Power Exchange in order to predict its production and payment assignments for any given cost/willingness-to-pay profile.

Using all this, agent i can obtain the expected production and profit for any possible

asking price. Then agent *i* selects the asking price that maximizes its expected profit and transmits this information to the System Operator & Power Exchange. This process is repeated a predefined number of times.

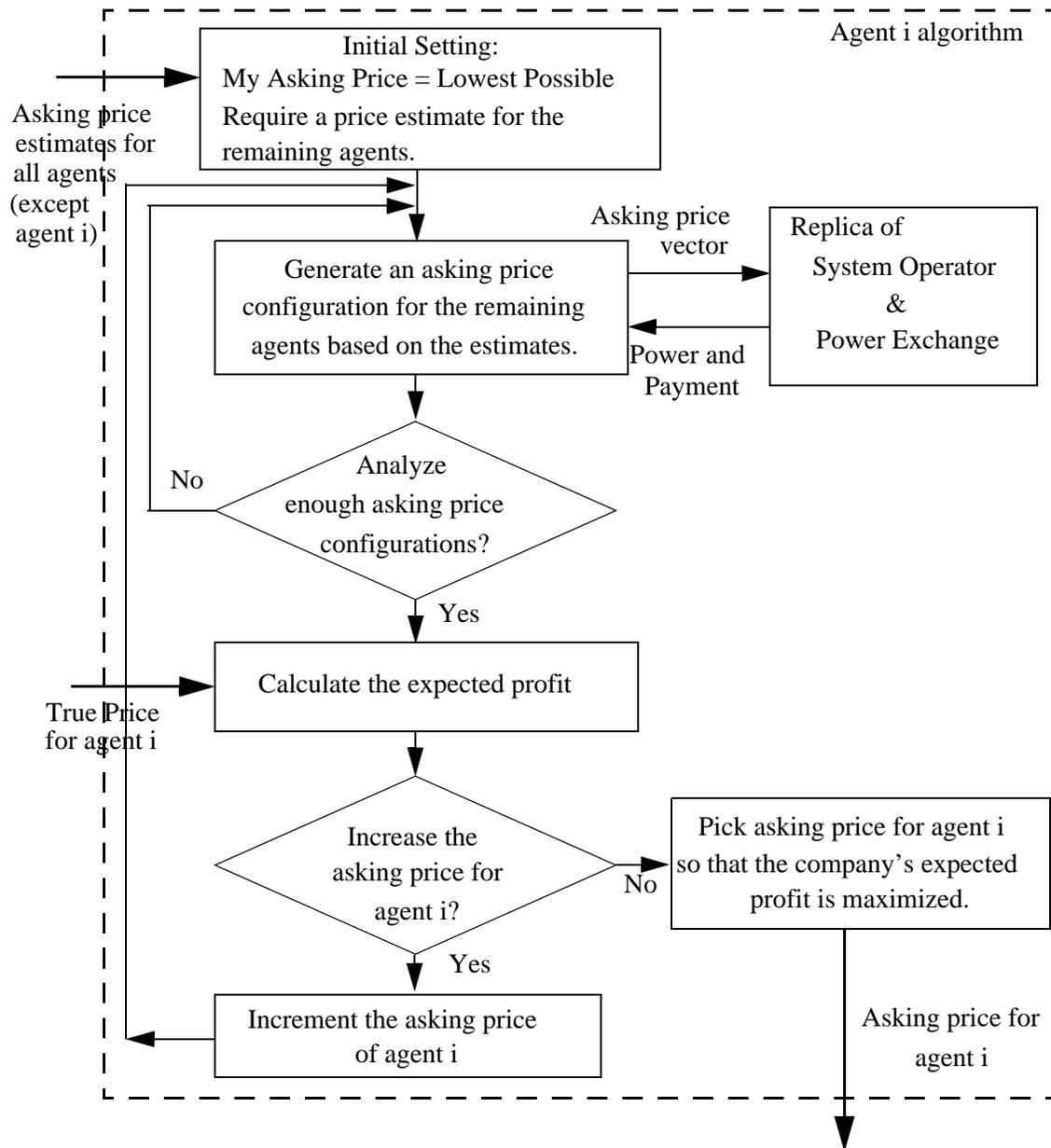
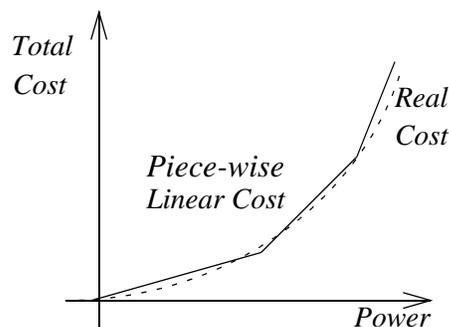


Figure 6-10: Agent *i* Algorithm

## 6.6 Multi-segment Agent Description

This subsection explains the possibility of representing the objective function of the dispatch using piece-wise linear functions. This technique has been shown to be useful when any of the following conditions makes it necessary to include a more complete representation the agent costs (generators) or willingness to pay (loads).

- The total cost of a generator or the willingness to pay of a consumer cannot be represented with a linear function. In this case a piece-wise linear function can better represent a wider variety of curves. The following figure shows an example of a generator total cost being represented with a piece-wise linear cost curve consisting of three segments.



**Figure 6-11: Real and Piece-wise Linear Total Cost for a Generator**

- A given company has more than one unit, either at one bus or at different buses.

The drawback of this technique is the time required to process a system using multi-segment curves. The details of the payment function for the multi-segment technique are given in the Appendix B.

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## CHAPTER 7 RESULTS - AGENT BID OPTIMIZATION

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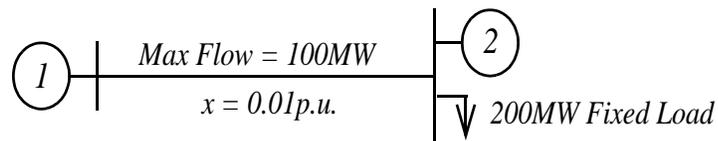
This chapter demonstrates the merits of the incentive compatible mechanism by means of a single period simulation on a 2-bus network and the IEEE-14-bus network. As it was described in subsection 6.3, an agent bid optimization can approximate the probability distribution of a generator's marginal cost by a finite grid, defining a set of possible costs, each with an associated probability. For each combination of marginal costs across the generators, the algorithm calculates the corresponding optimal power flow, including the total operating cost and an information compensation payment from eq. (5.4.2). These give an approximate means to calculate a generator's expected profit as a function of its submitted cost. The simulation results show that the incentive compatible mechanism is efficient whether the transmission network suffers a capacity shortage or not.

### **7.1 2-bus Case**

A 2-bus system is the simplest case to consider the burden of congestion in a transmission system. In this case a maximum of 100 MW flow between the buses is included to allow the possibility of congestion.

### 7.1.1 Description of the System

The data for this case is shown in Figure 7-1 and Table 7-1.



**Figure 7-1: 2-bus Case**

**Table 7-1: Generator Data for the 2-bus Case**

Generator (by location)	True Cost [\$/MWh]	Maximum Production [MW]	Minimum Production [MW]
1	28	150	0
2	18	300	0

In this case, each generator company, while knowing its true cost, estimates the costs of its competitors by drawing from a truncated normal distribution (between  $C_{\min}$  and  $C_{\max}$ ) representing each competitors costs.

Table 7-2 gives the generator cost probability distributions.

**Table 7-2: Probability Distribution Data for the 2-bus Case**

Generator (by location)	$C_{\min}$ [\$/MWh]	$C_{\max}$ [\$/MWh]	Mean [\$/MWh]	Standard Deviation
1	20	38	30	3
2	15	30	20	4

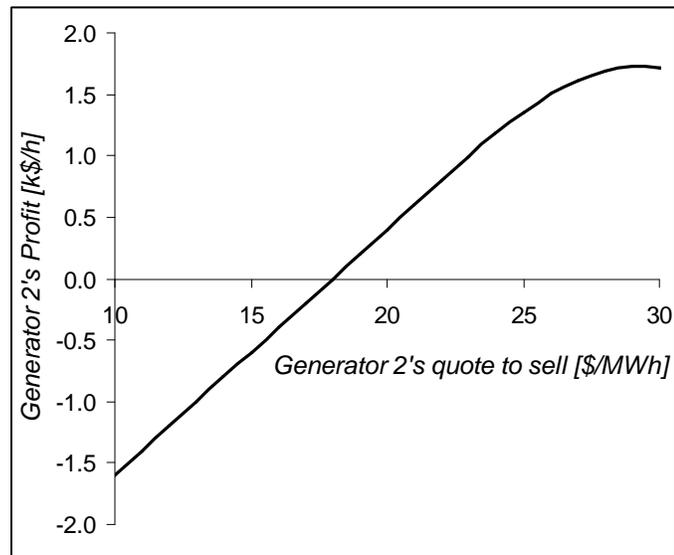
Since the transmission line between 1 and 2 allows only 100 MW there is a potential problem of congestion. Generator company 2 is said to have *market power*. Thus, left alone, generator company 2 would exaggerate (raise) its marginal cost in order to raise its expected revenue.

From generator 2's perspective, it is going to quote a marginal cost that maximizes its expected profit. Generator 2's revenue depends on the payment it gets for its power.

The traditional payment policy and the incentive compatible mechanism are contrasted in the following sections.

### **7.1.2 Traditional Mechanism**

As described in subsection 4.1, this mechanism simply pays the total cost (i.e. power times cost) quoted by a generator. Figure 7-2 plots generator 2's profit as a function of its quoted marginal cost.

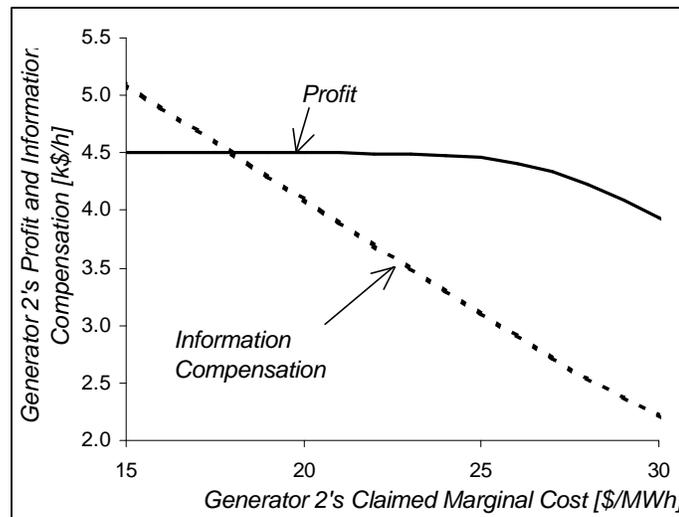


**Figure 7-2: Generator 2's Profit Using a *Traditional Mechanism***

As seen in Figure 7-2, due to its location, generator 2's optimal strategy is to quote its upper bound, 30 [\$/MWh]. Since generator 2 is distorting its real cost, the outcome of the economic dispatch is not likely to be efficient.

### 7.1.3 Incentive Compatible Mechanism

On the other hand, the incentive compatible mechanism (described in Chapter 5) includes an extra payment that would correct generator 2's expected profit function. This payment eliminates any gain from overstating one's marginal cost (in this case 18 [\$/MWh]). Figure 7-3 plots both this extra payment and the resulting profit function for generator 2.



**Figure 7-3: Generator 2's Profit and Information Compensation Using the Incentive Compatible Mechanism**

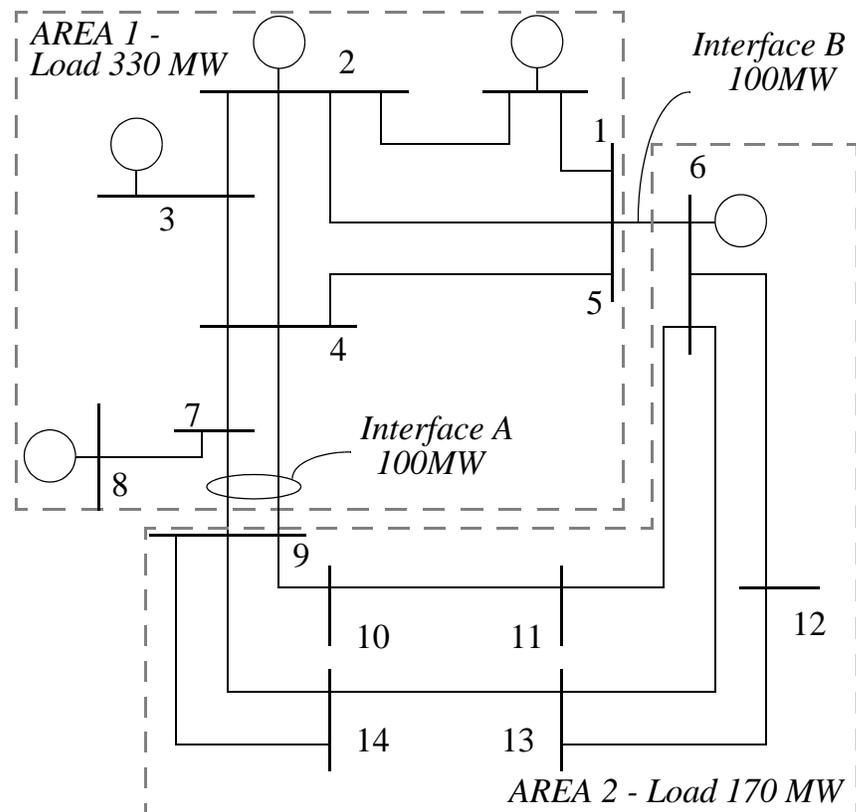
The new profit function is at its maximum at generator 2's true cost (18 [\$/MWh]), therefore, efficiency can be expected. In this case there is a wide range of marginal cost that maximize generator 2's profit, but it still has no incentive to distort its true cost.

## 7.2 IEEE-14-bus Case

As a second example a slightly modified version of the IEEE-14-bus case was used. This system includes 21 transmission lines and 5 generators.

### 7.2.1 Description of the System

The system was divided into two areas, as shown in Figure 7-4. The capacity of each interface between the two areas (A and B) is 100 MW.



**Figure 7-4: IEEE-14-bus Case**

The generator data is included in Figure 7-4 and Table 7-3.

**Table 7-3: Generator Data for the IEEE-14-bus Case**

Generator (by location)	True Cost [\$/MWh]	Minimum Production [MW]	Maximum Production [MW]
1	4.5	0	600
2	8.0	0	600
3	4.0	0	600
6	6.0	0	600
8	8.0	0	600

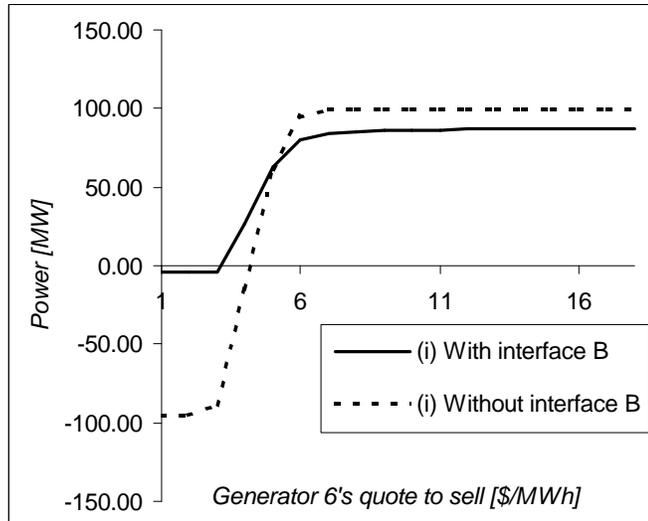
Again in this case, every generator company, while knowing its true cost, estimates the costs of its competitors by drawing from a truncated normal distribution (between  $C_{\min}$  and  $C_{\max}$ ). Table 7-4 gives the generator cost probability distributions.

**Table 7-4: Probability Distribution Data for the IEEE-14-bus Case**

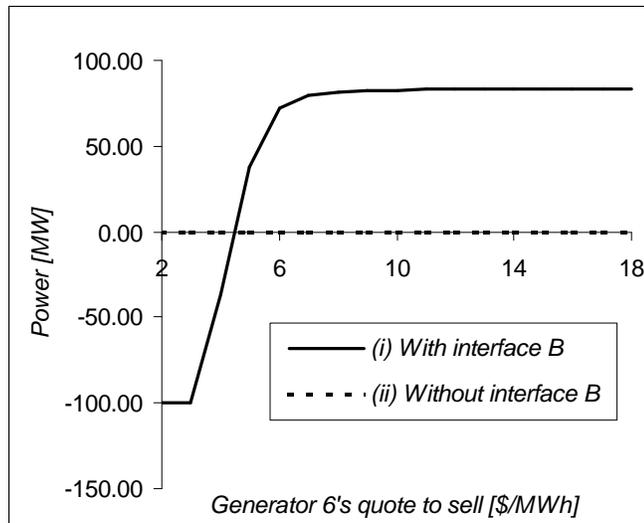
Generator (by location)	$C_{\min}$ [\$/MWh]	$C_{\max}$ [\$/MWh]	Mean [\$/MWh]	Standard Deviation
1	3.0	8.0	5.0	2.0
2	6.0	10.0	9.0	2.0
3	3.0	6.0	5.0	1.0
6	2.0	18.0	5.5	3.0
8	5.0	9.0	8.0	1.0

These simulations focus on the behavior of generator 6 in two scenarios: (i) interfaces A and B operating, so there is enough transfer capacity between area 1 and area 2, so

that generator 6 must compete in the market without market power, (ii) only interface A is operating, giving generator 6 the opportunity to be a local monopoly with market power. The interface flows can be seen in the next two figures.

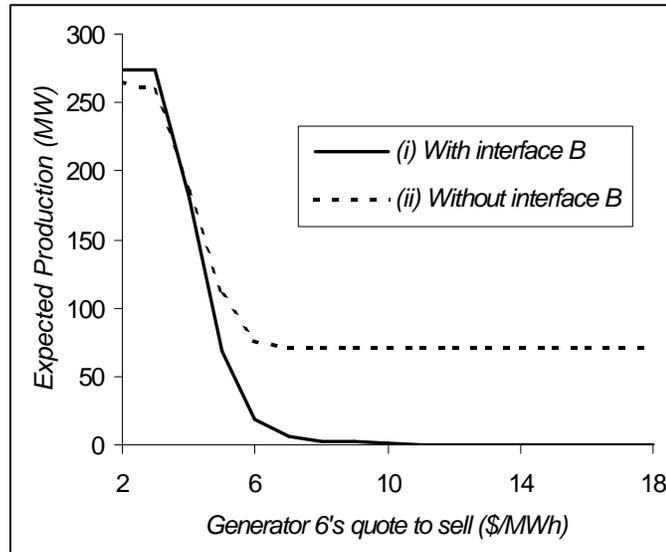


**Figure 7-5: Interface A Flow vs. Generator's 6 Quote to Sell**



**Figure 7-6: Interface B Flow vs. Generator's 6 Quote to Sell**

The expected production vs. generator 6's quote to sell can be seen in Figure 7-7 for both scenarios. Notice that in scenario (ii), generator 6 is dispatched "out-of-merit" for 70 MW due to congestion in interface A.



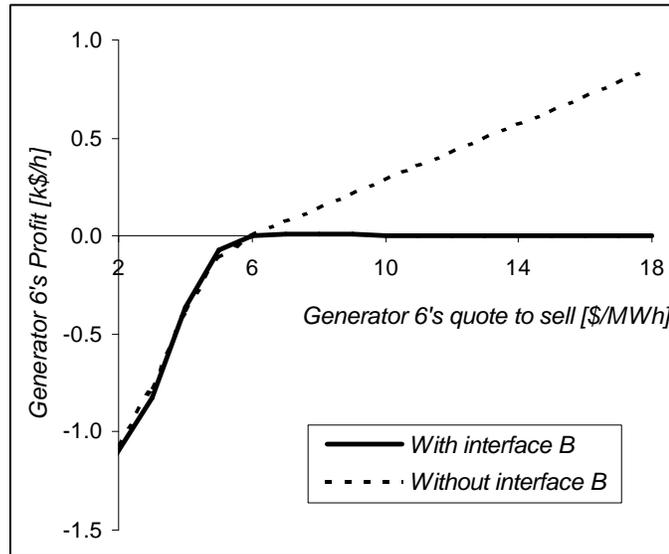
**Figure 7-7: Generator 6's Expected Production vs. its Quote to Sell**

As always, this company is going to quote the cost that maximizes its expected profit when asked to provide a marginal cost, since generator 6's revenue depends on the payment it gets for its production. Below is the comparison of the traditional with the incentive compatible mechanism in this setting for the two scenarios.

### 7.2.2 Traditional Mechanism

As previously described, this mechanism simply pays the total cost quoted by a generator times its MW production. Figure 7-8 shows generator 6's profit as a function

of its quoted marginal cost.



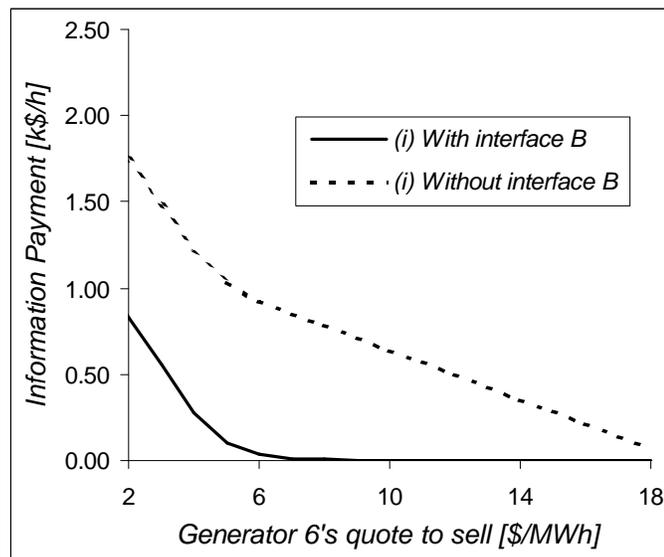
**Figure 7-8: Generator 6's Profit vs. its Quote to Sell Using a *Traditional Mechanism***

As seen in Figure 7-8, in the first scenario generator 6's optimal strategy is to slightly exaggerate its marginal cost to 7 [\$/MWh]. The problem is that, quoting around 7 [\$/MWh], generator 6 is going to be assigned only one third of its corresponding economic dispatch production. In the second scenario generator 6's optimal strategy is to exaggerate its marginal cost to the highest level, 18 [\$/MWh]. Again causing its assignment to shift 5MW from the optimum (economic dispatch).

In both scenarios, since generator 6 is distorting its real cost, the outcome of the economic dispatch is not efficient. Notice that without interface B generator 6 is in a better position due to its monopoly status.

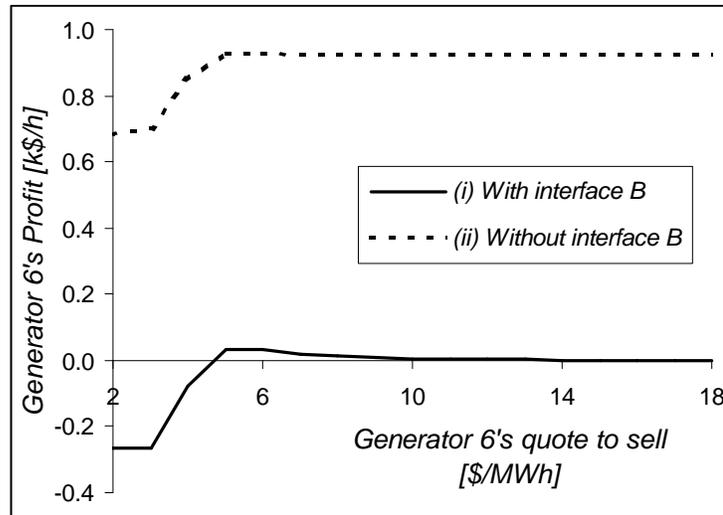
### 7.2.3 Incentive Compatible Mechanism

The mechanism described in Chapter 5 includes an extra payment that would correct generator 6's expected profit function. This payment eliminates any gain from overstating one's marginal cost, i.e., quoting a cost other than 6 [\$/MWh]. In the following figure this information payment can be seen for both cases with and without interface B.



**Figure 7-9: Generator 6's Information Payment vs. its Quote to Sell Using the Incentive Compatible Mechanism**

Figure 7-10 shows the resulting profit curves for both proposed scenarios. Figure 7-10 shows that generator 6's best choice is to quote its true cost, since its profit is maximum at that cost.



**Figure 7-10: Generator 6's Profit vs. its Quoted Marginal Cost Using the Incentive Compatible Mechanism**

The same argument applies to other generators, and efficiency is achieved.

Thus, whether interface B is operative or not, the incentive compatible mechanism is efficient and the traditional mechanism is inefficient. Furthermore, the inefficiency of the traditional mechanism is more severe when interface B is not operative, since a shortage in transmission capacity increases the monopoly power of some generator companies.

### 7.3 Multi-Segment Simulation

As described in subsection 6.6 one of the important improvement of the incentive compatible mechanism described in Chapter 5 is the representation of a generator

company total cost or a load willingness to pay function using a multi-segment curve instead of a single straight line. Often participant costs (generators) or willingness to pay (loads) functions are not linear. In those cases a piece-wise linear function can better represent non-linearities of the participant's objective function. The technique can also be used to represent the case of generator companies owning machines located at different buses and load companies having consumers at more than one bus. As mentioned before, the only disadvantage of this technique is the time required to process a system.

This subsection presents the same case used in subsection 7.2 while representing generator company 6 as a unit whose cost is represented using two linear segments instead of one.

### **7.3.1 Description of the System**

The data for the cost function of the generators is given in the following table (no load company was include in this simulation). A diagram of this system can be see in subsection 7.2.

**Table 7-5: Generator Data for the IEEE-14-bus Case**

Generator (by location)	True Cost [\$/MWh]	Minimum Production [MW]	Maximum Production [MW]
1	11.5	0	600
2	10.2	0	600
3	10.7	0	600
6a	10.3	0	300
6b	10.7	0	300
8	12.8	0	600

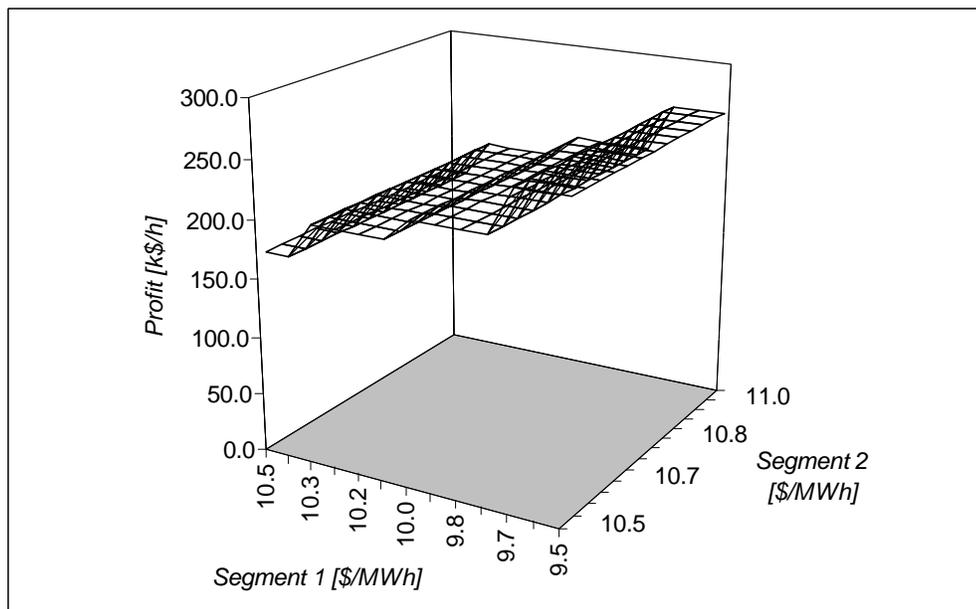
Again in this case, every generator company, while knowing its true cost, assumes the costs of its competitors are estimated by drawing from a truncated normal distribution (between  $C_{\min}$  and  $C_{\max}$ ). Table 7-6 shows the generator cost probability distributions.

**Table 7-6: Probability Distribution Data for the IEEE-14-bus Case**

Generator (by location)	$C_{\min}$ [\$/MWh]	$C_{\max}$ [\$/MWh]	Mean [\$/MWh]	Standard Deviation
1	10.5	13.0	11.5	1.0
2	9.0	11.0	10.0	1.0
3	10.0	11.5	11.0	1.0
6a	9.5	10.5	10.5	1.0
6b	10.2	11.0	10.8	0.6
8	12.0	14.0	13.0	1.0

As shown in the tables above, generator 6 has two segment describing its total cost. As before, generator company 6 is going to quote costs that maximize its expected profit.

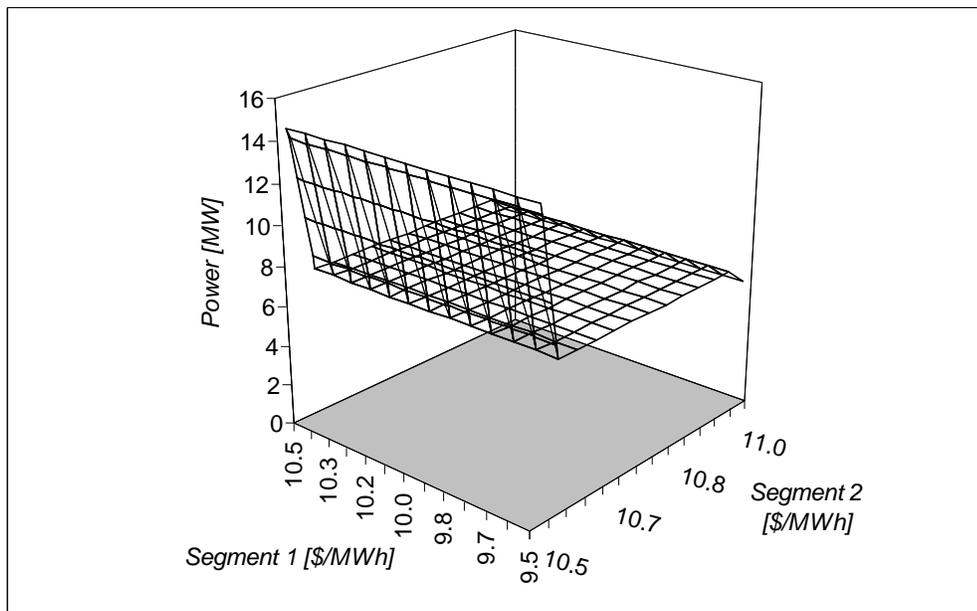
To obtain the optimal cost to quote, each generator company calculates the expected production for any possible cost it may quote. The following figure shows the expected generation of the first segment of the company (the lower cost). It can be appreciated that this segment produces its maximum when its quote is the minimum possible (9.5 [\$/MWh]) and it does not depend much on what is quoted for segment 2.



**Figure 7-11: Generation in Segment 1**

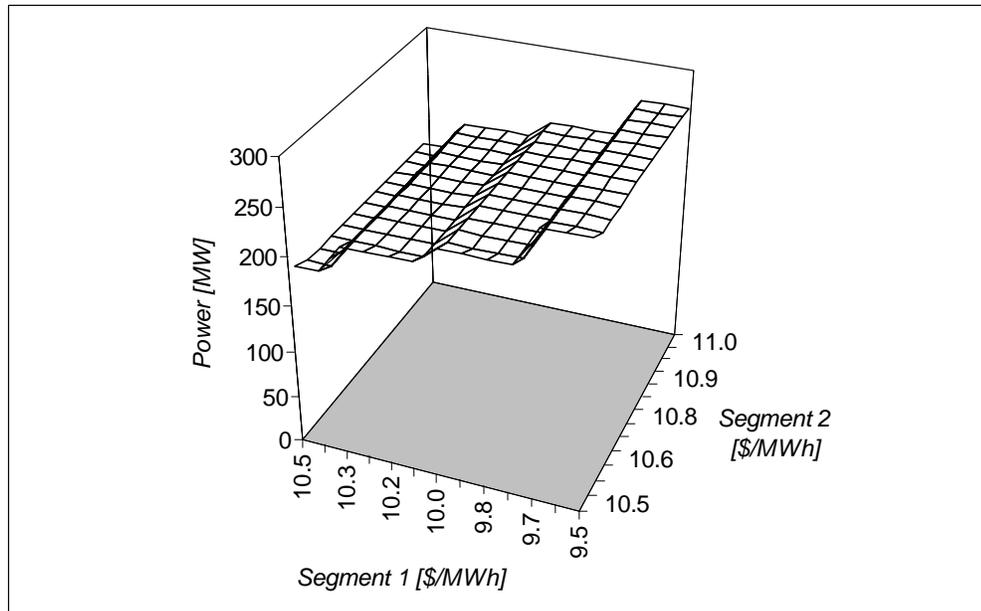
The next figure shows the expected generation of the second segment of the company (higher cost). This segment produces its maximum when its quote is the minimum

possible (10.5 [\$/MWh]) and does not depend on what was quote by segment 1. Either way, since segment 2 is less efficient it produces much less than segment 1.



**Figure 7-12: Generation in Segment 2**

Finally, the next figure shows the aggregate production of generator company 6 for all quotes of segment 1 and segment 2.

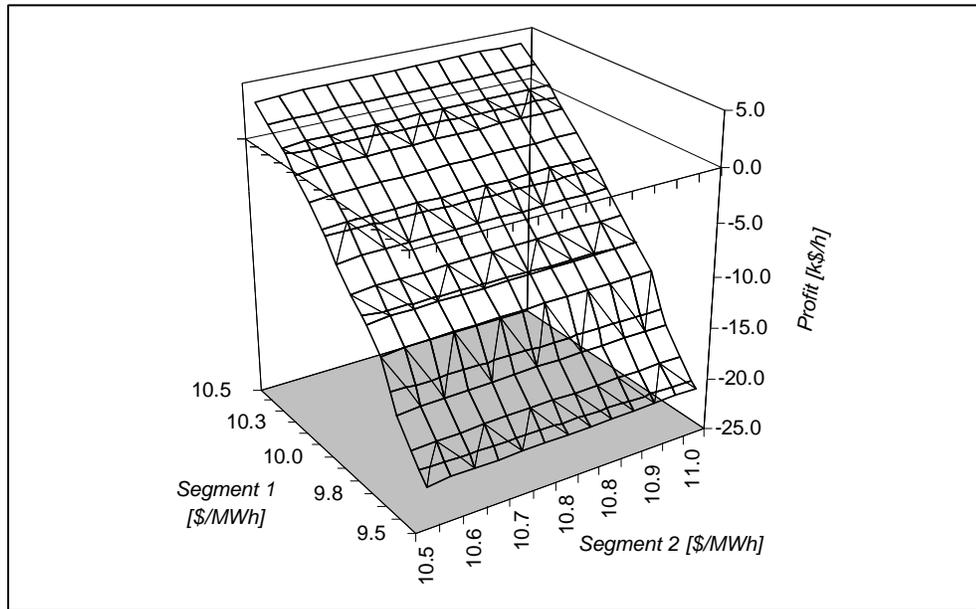


**Figure 7-13: Aggregate Production of Generator Company 6**

As always, generator 6's revenue depends on the payment it gets for its power, so below are contrasted a traditional payment policy and incentive compatible mechanism. Generator company 6 must go to calculate its expected power assignment for its two sub-units (two segments).

### 7.3.2 Traditional Mechanism

As described in subsection 4.1, this mechanism simply pays the total cost (i.e. power times cost) quoted by a generator. Figure 7-14 shows generator 6's profit as a function of the quotes provided for each one of its segments in this traditional mechanism.



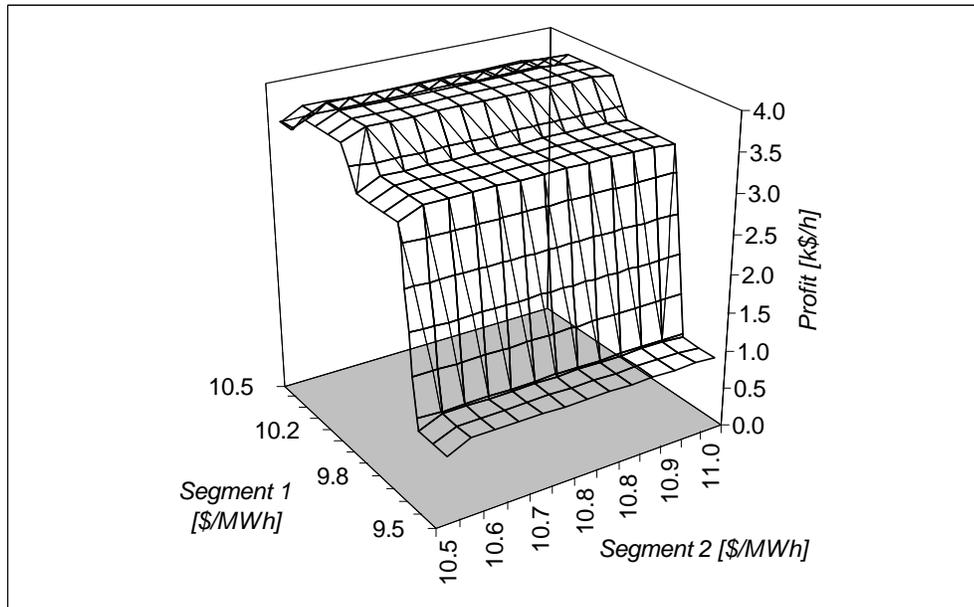
**Figure 7-14: Generator 6's Profit Using a *Traditional* Mechanism**

As seen in Figure 7-14, generator 6's optimal strategy is to quote its upper bound, 10.5 [\$/MWh] for segment 1 and 11.0 [\$/MWh] for segment 2. Since generator company 6 is distorting its true cost, the outcome of the economic dispatch is not likely to be efficient.

### 7.3.3 Incentive Compatible Mechanism

On the other hand, the incentive compatible mechanism includes an extra payment that would correct generator 6's expected profit function. This payment eliminates any gain from overstating its marginal cost for any of its segments (in this case 10.3 [\$/MWh] for segment 1 and 10.7 [\$/MWh] for segment 2).

In Figure 7-15 it can be seen both this extra payment and the resulting profit function for generator 2.



**Figure 7-15: Generator 6's Profit and Information Compensation Using the Incentive Compatible Mechanism**

The new profit function has a maximum at generator 6's true cost, therefore, efficiency can be expected. In this case there is a wide range of marginal cost that maximize generator 6's profit (the intersection of the intervals 10.0833 - 10.333 [\$/MWh] for segment 1 and 10.54 - 10.96 [\$/MWh] for segment 2), but it still has no incentive to distort its true cost for its two segments.

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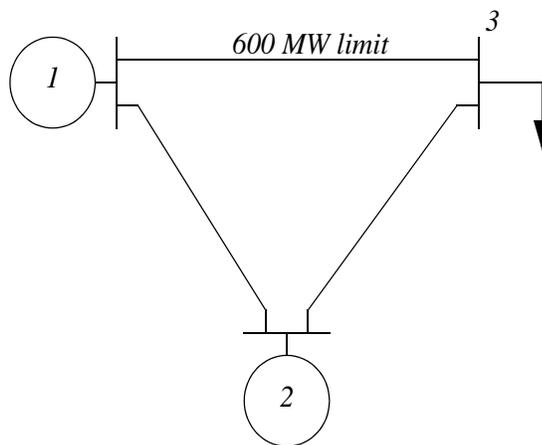
## CHAPTER 8 RESULTS - MULTI-PERIOD MODEL

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In this chapter the different proposals shown in Chapter 4 are compared using the multi-period simulation described in subsection 6.5.

### 8.1 Study Case

The system shown in the following figure has 3 lines and 3 buses. The lines are identical, with the exception of line 1-3 that may or may not have a 600 MW limit (both cases with and without the limit are going to be simulated and compared).



**Figure 8-1: 3-bus System**

The limits and the true costs of the generators and the true willingness to pay of the consumer are shown in the following table.

**Table 8-1: Cost (Generators) and Willingness to Pay (Consumers)**

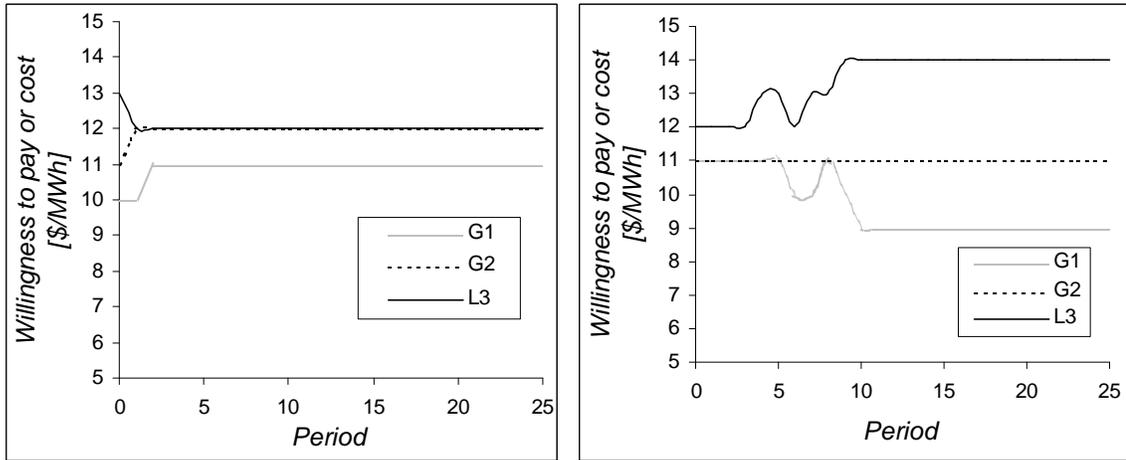
Agent (generator or consumer by location)	True Cost (generators) and willingness to pay (consumers)	Minimum [MW]	Maximum [MW]
1	6	0	2000
2	10	0	2000
3	21	900	2900

## 8.2 Results

This subsection illustrates the results of the simulation using each one of the payment schemes described in Chapter 4. The horizon was set to 25 periods, which is sufficient for each scheme converge to a stable set of quotes. As in the previous chapter, every agent estimates the costs (or willingness to pay) of its competitors by drawing from a truncated normal distribution. Using each payment scheme a simulation is analyzed with no transmission limits and another with the 600-MW limit on line 1-3 (from subsection 8.2.1 to 8.2.4).

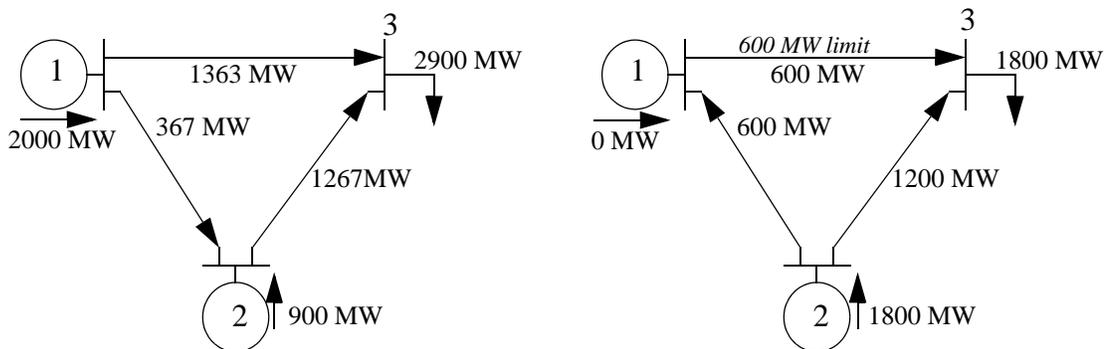
### 8.2.1 Traditional Payment Scheme

In the case with no transmission limits the quotes show an immediate convergence to 11, 12 and 12 [\$/MWh] for generator 1, generator 2 and load 3, respectively. On the other hand, the case with a 600 MW transmission limit on line 1-3, the quotes converge to 9, 11 and 14 [\$/MWh] for generator 1, generator 2 and load 3, respectively.



**Figure 8-2: Traditional Payment with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

The equilibrium quotes in both situations differ demonstrating that congestion affects the way the participants perceive the market. With the transmission limit, generator company 1 decreases its quote, but none of its energy is purchased, while generator company 2 sells 900 MW more, and decreases its quote by only 1 [\$/MWh]. These results can be seen in the following figure.



**Figure 8-3: Equilibrium for the Traditional Payment with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

Notice that even with generation company 1 quoting 2 units less than generator company 2, it is optimum for the load to only buy from generator 2 because it can buy 1800 MW. (If it buys from generator 1, the maximum power that can be transferred to 3 would be 900 MW.)

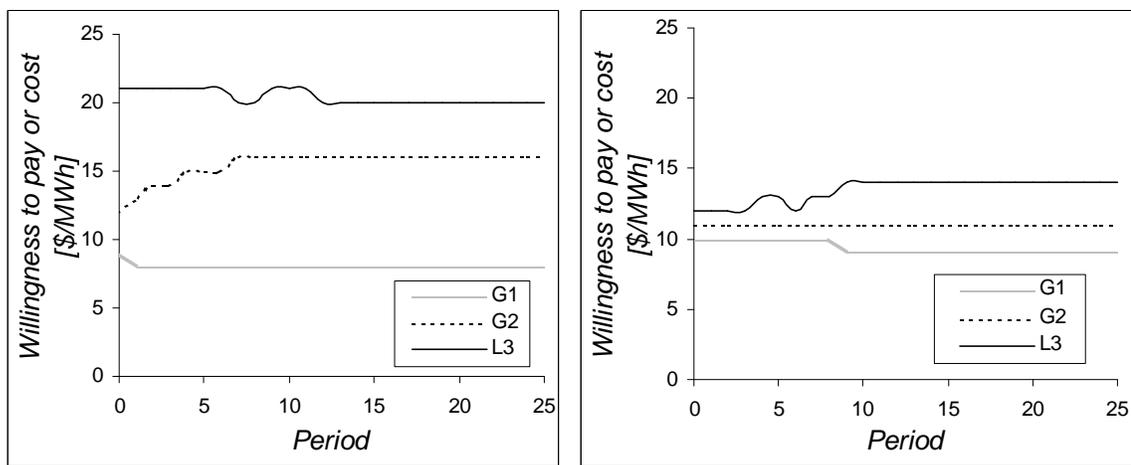
**Table 8-2: Equilibrium for the Traditional Payment with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

	No transmission limits	600-MW limit on line 1-3
Generator 1	Quote to sell: 11 [\$/MWh] True cost: 6 [\$/MWh] Power: 2000 [MW] Revenue received: 22 [k\$/h] Profit: 10 [k\$/h]	Quote to sell: 9 [\$/MWh] True cost: 6 [\$/MWh] Power: 0 [MW] Revenue received: 0 [k\$/h] Profit: 0 [k\$/h]
Generator 2	Quote to sell: 12 [\$/MWh] True cost: 10 [\$/MWh] Power: 900 [MW] Revenue received: 10.8 [k\$/h] Profit: 1.8 [k\$/h]	Quote to sell: 11 [\$/MWh] True cost: 10 [\$/MWh] Power: 1800 [MW] Revenue received: 19.8 [k\$/h] Profit: 1.8 [k\$/h]
Load 3 - Variable	Quote to buy: 12 [\$/MWh] Willingness to pay: 21 [\$/MWh] Power: 2000 [MW] Payment: 24 [k\$/h] Profit: 18 [k\$/h]	Quote to buy: 14 [\$/MWh] Willingness to pay: 21 [\$/MWh] Power: 900 [MW] Payment: 12.6 [k\$/h] Profit: 6.3 [k\$/h]
Load 3 - Fixed	Power: 900 [MW]	Power: 900 [MW]

With or without the line limit, the participants quoted very different values than their true cost or true willingness to pay. Due to these distortions in the quotes, the resulting allocation of power is not efficient in either case.

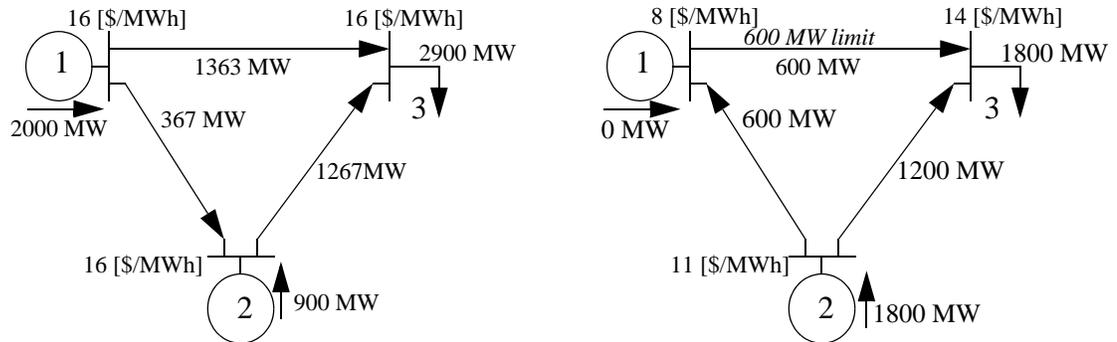
### 8.2.2 Bus Marginal Cost Payment Scheme

In the case with no transmission limit the quotes show a convergence to the quotes 8, 16 and 20 [\$/MWh] for generator 1, generator 2 and load 3, respectively, whereas the case with a 600-MW-transmission limit on line 1-3, the quotes converge to 9, 11 and 14 [\$/MWh] for generator 1, generator 2 and load 3, respectively.



**Figure 8-4: Bus Marginal Cost Payment with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

The equilibrium quotes in both situations differ demonstrating that congestion affects the way the participant behave. With the transmission limitation, load 3 decreases its willingness to pay by 6 units, and still receives 1800 MW. These results can be seen in the following figure.



**Figure 8-5: Equilibrium for the Bus Marginal Cost Payment with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

Notice that even with generation company 1 quoting 2 units less than generator company 2, it is optimum for the load to buy from generator 2 because it can buy 1800 MW. (If it buys from generator 1, the maximum power that can be transfer to 3 would be 900 MW.)

**Table 8-3: Equilibrium for the Bus Marginal Cost Payment with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

	No transmission limits	600-MW limit on line 1-3
Generator 1	Quote to sell: 8 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 2000 [MW] Revenue received: 32 [k\$/h] Profit: 20 [k\$/h]	Quote to sell: 9 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 8 [\$/MWh] Power: 0 [MW] Revenue received: 0 [k\$/h] Profit: 0 [k\$/h]
Generator 2	Quote to sell: 16 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 900 [MW] Revenue received: 14.4 [k\$/h] Profit: 5.4 [k\$/h]	Quote to sell: 11 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 11 [\$/MWh] Power: 1800 [MW] Revenue received: 19.8 [k\$/h] Profit: 1.8 [k\$/h]
Load 3 - Variable	Quote to buy: 20 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 2000 [MW] Payment: 32 [k\$/h] Profit: 10 [k\$/h]	Quote to buy: 14 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus marginal cost: 14 [\$/MWh] Power: 900 [MW] Payment: 12.6 [k\$/h] Profit: 6.3 [k\$/h]
Load 3 - Fixed	Power: 900 [MW]	Power: 900 [MW]

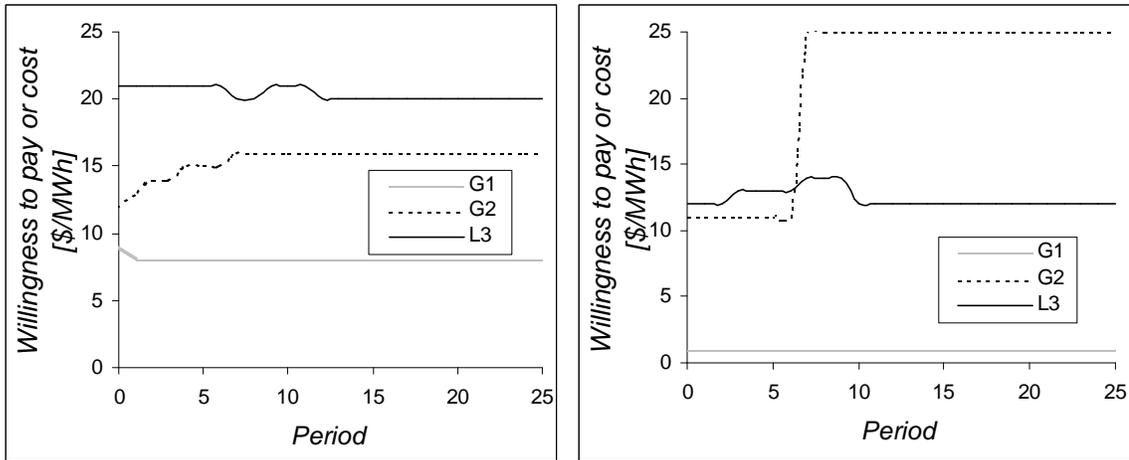
With or without the line limit, the participants quote very different values than their true cost or true willingness to pay. Due to these distortions in the quotes, the resulting allocation of power is not efficient.

### **8.2.3 Transmission Congestion Contract Payment Scheme**

In this subsection four different cases of transmission congestion contracts are included. In the first, generator company 1 owns a 900-MW transmission congestion contract for transmission from 1 to 3. In the second, generator company 2 owns an 1800-MW transmission congestion contract for transmission from 2 to 3. The third case assumes that the load company owns a 900-MW transmission congestion contract for transmission from 1 to 3, and finally in the fourth case, the load company is considered to own an 1800-MW contract for transmission from 2 to 3. In all these cases the contract is assumed to have been purchased in advance, either by a generator company or the load company in a secondary market.

#### **Case 1 - Generator Company 1 Owns a Transmission Congestion Contract for Transmission from 1 to 3**

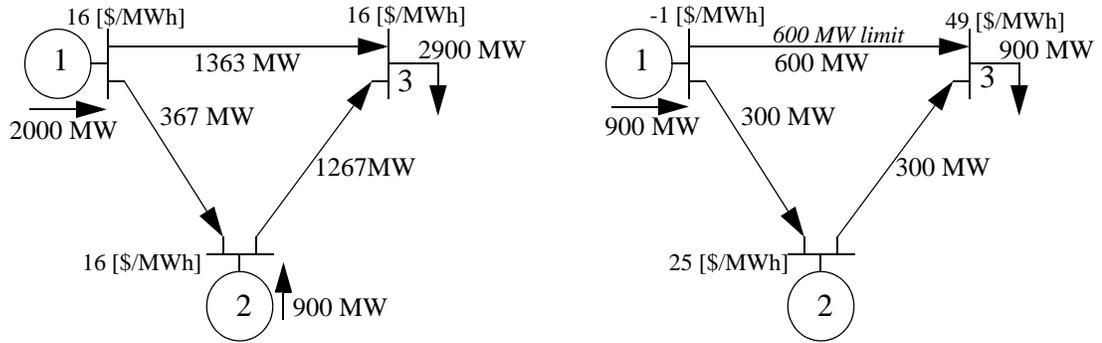
In the case with no transmission limits the quotes converge to 8, 16 and 20 [\$/MWh] for generator 1, generator 2 and load 3, respectively. On the other hand, the case with a transmission limit on line 1-3, the quotes converge to 1, 12 and 25 [\$/MWh] for generator 1, generator 2 and load 3, respectively. These results are displayed in the following figure.



**Figure 8-6: Transmission Congestion Contract (Generator Company 1) with no Transmission Limit (left), and with a 600-MW Limit on line 1-3 (right)**

Notice that applying a transmission congestion contract scheme with no congestion produces the same results as the bus marginal cost payment scheme (see Figure 8-5), this is due to the fact that the contract will be useful to generator company 1 only when congestion arises (making marginal prices differ from one bus to another). In the case with the transmission limit, generator company 1 is willing to quote a very low cost (1 [\$/MWh]) just with the objective of overloading the network and causing congestion, and with congestion, it obtains more profit from its transmission congestion contract.

The generations, loads, and line flows for both situations are shown in the following figure.



**Figure 8-7: Equilibrium for Transmission Congestion Contract (Generator Company 1) with no Transmission Limit (left), and with a 600-MW Limit on line 1-3 (right)**

The contract in this case will pay 30 [k\$/h] to generator company 1 due to the high difference in marginal cost between buses 1 and 3. (A peculiar feature of this system is the negative sign of the marginal cost at bus 1, this implies that the system will actually save money if the load is increased at that bus.)

All the details of the equilibrium are include in the following table (profits, marginal costs, payments, TCC payments, etc.).

**Table 8-4: Equilibrium for Transmission Congestion Contract (Generator Company 1) with no Transmission Limit (left), and with a 600-MW Limit on line 1-3 (right)**

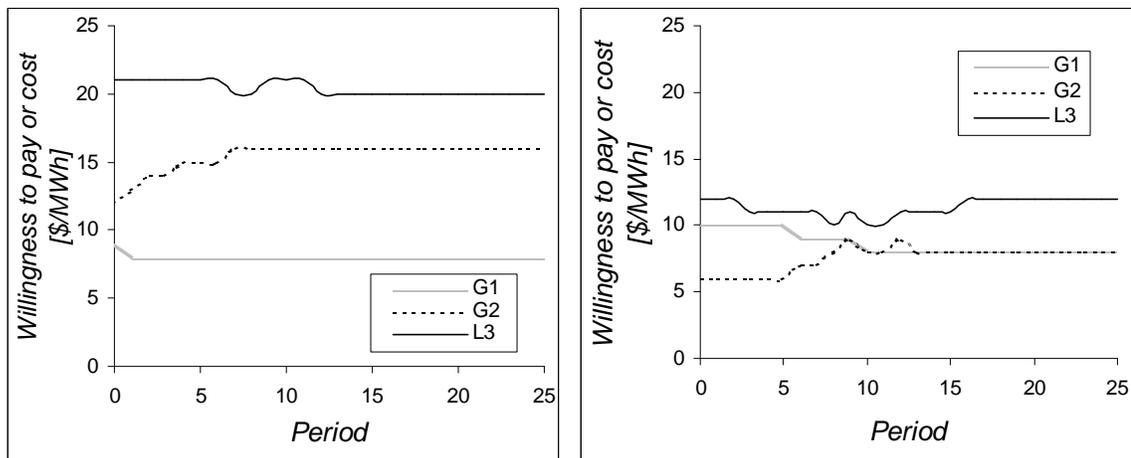
	No transmission limits	600-MW limit on line 1-3
Generator 1	Quote to sell: 8 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 2000 [MW] TCC payment: 0 [k\$/h] Revenue received: 32 [k\$/h] Profit: 20 [k\$/h]	Quote to sell: 1 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: -1 [\$/MWh] Power: 900 [MW] TCC payment: 45 [k\$/h] Revenue received: 45.9 [k\$/h] Profit: 0 [k\$/h]
Generator 2	Quote to sell: 16 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 900 [MW] Revenue received: 14.4 [k\$/h] Profit: 5.4 [k\$/h]	Quote to sell: 25 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 25 [\$/MWh] Power: 0 [MW] Revenue received: 0 [k\$/h] Profit: 0 [k\$/h]
Load 3 - Variable	Quote to buy: 20 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus Marginal Cost: 16 [\$/MWh] Power: 2000 [MW] Payment: 32 [k\$/h] Profit: 10 [k\$/h]	Quote to buy: 12 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus Marginal Cost: 49 [\$/MWh] Power: 0 [MW] Payment: 0 [k\$/h] Profit: 0 [k\$/h]
Load 3 - Fixed	Power: 900 [MW]	Power: 900 [MW]

With or without the line limit, the participants quote very different values than their true cost or true willingness to pay. Due to these distortions in the quotes, the resulting allocation of power is not efficient. Moreover, in this case the solution obtained by the OPF is clearly non-optimal; if the true cost values are considered (the more efficient

result), the total benefit of the system decreases from 19.8 k\$ to 13.5 k\$ per hour (around 46%). This clearly shows that the inclusion of the transmission congestion contract can send very misleading signals to the participants in the market (the distortion from the optimum are far from the previous payment schemes).

### Case 2 - Generator Company 2 Owns a Transmission Congestion Contract for Transmission from 2 to 3

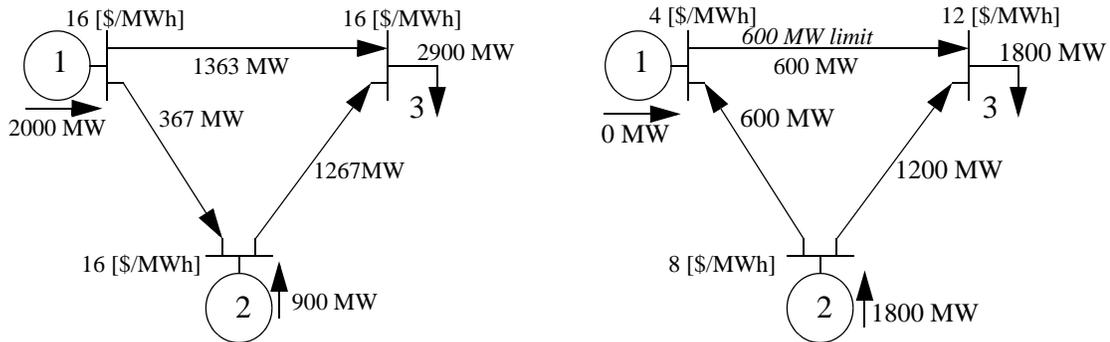
In this case generator company 2 owns the transmission congestion contract for the transmission from 2 to 3. In the case with no transmission limits the quotes converge to the quotes 8, 16 and 20 [\$/MWh] for generator 1, generator 2 and load 3, respectively. On the other hand, the case with a 600-MW transmission limit on line 1-3, the quotes converge to 8, 8 and 12 [\$/MWh] for generator 1, generator 2 and load 3, respectively. These results are displayed in the following figure.



**Figure 8-8: Transmission Congestion Contract (Generator Company 2) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

Notice that applying a transmission congestion contract scheme with no congestion produces the same results as the bus marginal cost payment scheme (see Figure 8-5), this is due to the fact that the contract will be useful to generator company 2 only when congestion arises (making marginal prices differ from one bus to another). In the case with the transmission limit, generator company 2 is willing to quote a cost 2 [\$/MWh] which is much less than its true cost of 8 [\$/MWh] with the objective of causing congestion in the network so as to obtain more profit from its transmission congestion contract payment.

The generations, loads, and line flows for both situations are shown in the following figure.



**Figure 8-9: Equilibrium for Transmission Congestion Contract (Generator Company 2) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

The contract in this case will pay 7.2 [k\$/h] to generator company 2 due to the

difference in marginal cost between buses 2 and 3. All the details of the equilibrium are included in the following table (profits, marginal costs, payments, TCC payments, etc.).

**Table 8-5: Equilibrium for Transmission Congestion Contract (Generator Company 2) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

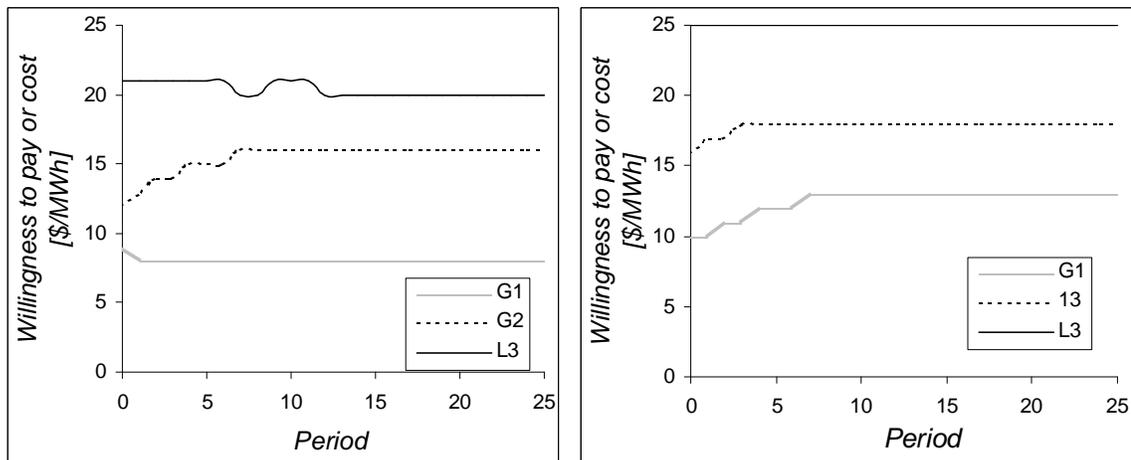
	No transmission limits	600-MW limit on line 1-3
Generator 1	Quote to sell: 8 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 2000 [MW] Revenue received: 32 [k\$/h] Profit: 20 [k\$/h]	Quote to sell: 8 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 4 [\$/MWh] Power: 0 [MW] Revenue received: 0 [k\$/h] Profit: 0 [k\$/h]
Generator 2	Quote to sell: 16 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 900 [MW] TCC payment: 0 [k\$/h] Revenue received: 14.4 [k\$/h] Profit: 5.4 [k\$/h]	Quote to sell: 8 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 8 [\$/MWh] Power: 1800 [MW] TCC payment: 7.2 [k\$/h] Revenue received: 21.6 [k\$/h] Profit: 7.2 [k\$/h]
Load 3 - Variable	Quote to buy: 20 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 2000 [MW] Payment: 32 [k\$/h] Profit: 10 [k\$/h]	Quote to buy: 12 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus marginal cost: 12 [\$/MWh] Power: 900 [MW] Payment: 10.8 [k\$/h] Profit: 8.1 [k\$/h]
Load 3 - Fixed	Power: 900 [MW]	Power: 900 [MW]

With or without the line limit, the participants quote very different values than their true cost or true willingness to pay. Due to these distortions in the quotes, the resulting

allocation of power is not efficient. This again shows that the inclusion of the transmission congestion contract can send very misleading signals to the participant in the market.

### Case 3 - Load Company 3 Owns a Transmission Congestion Contract for Transmission from 1 to 3.

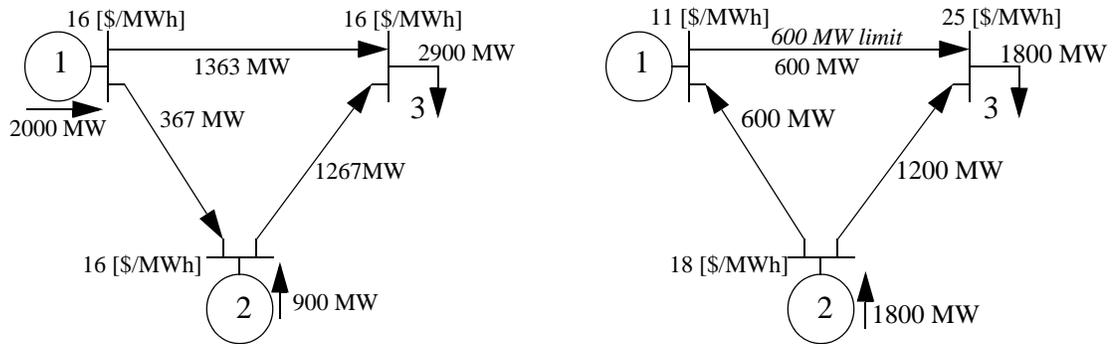
In this case the load company 3 owns the transmission congestion contract for the transmission from 1 to 3. In the case with no transmission limits the quotes converge to the quotes 8, 16 and 20 [\$/MWh] for generator 1, generator 2 and load 3, respectively. On the other hand, the case with a 600-MW-transmission limit on line 1-3, the quotes converge to 13, 18 and 25 [\$/MWh] for generator 1, generator 2 and load 3, respectively. These results are displayed in the following figure.



**Figure 8-10: Transmission Congestion Contract (Load Company 3) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

Notice that applying a transmission congestion contract scheme with no congestion produces the same results as the bus marginal cost payment scheme (see Figure 8-5), this is due to the fact that the contract will be useful to generator company 2 only when congestion arises (making marginal prices differ from one bus to another). In the case with the transmission limit, load company 3 is willing to quote 25 [\$/MWh] (4 [\$/MWh] more than its true cost of 21 [\$/MWh]) with the objective of causing congestion, and with that, obtaining more profit from its transmission congestion contract payment.

The generations, loads, and line flows for both situations are shown in the following figure.



**Figure 8-11: Equilibrium for Transmission Congestion Contract (Load Company 3) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

The contract in this case will pay 12.6 [k\$/h] to the load company due to the difference in marginal cost between buses 1 and 3 (900 MW times 25 - 11 = 14 [\$/MWh]). This amount is more than enough to cover the losses that the load company will incur for

increasing its willingness to pay above its true value, and still give the company a profit of 9 [k\$/h]. All the details of the equilibrium are include in the following table (profits, marginal costs, payments, TCC payments, etc.).

**Table 8-6: Equilibrium for Transmission Congestion Contract (Load Company 3) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

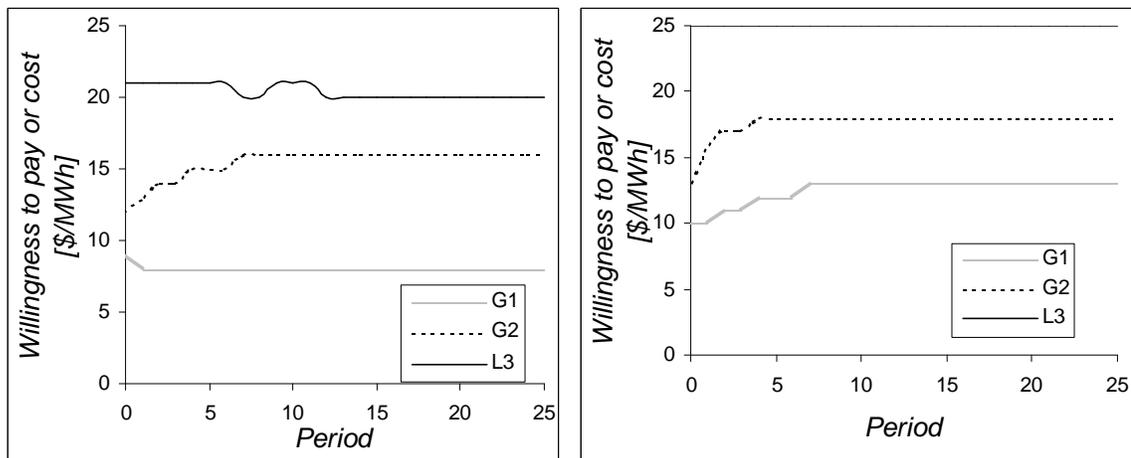
	No transmission limits	600-MW limit on line 1-3
Generator 1	Quote to sell: 8 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 2000 [MW] Revenue received: 32 [k\$/h] Profit: 20 [k\$/h]	Quote to sell: 13 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 11 [\$/MWh] Power: 0 [MW] Revenue received: 0 [k\$/h] Profit: 0 [k\$/h]
Generator 2	Quote to sell: 16 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 900 [MW] Revenue received: 14.4 [k\$/h] Profit: 5.4 [k\$/h]	Quote to sell: 18 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 18 [\$/MWh] Power: 1800 [MW] Revenue received: 32.4 [k\$/h] Profit: 14.4 [k\$/h]
Load 3 - Variable	Quote to buy: 20 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus Marginal Cost: 16 [\$/MWh] Power: 2000 [MW] TCC payment: 0 [k\$/h] Payment: 32 [k\$/h] Profit: 10 [k\$/h]	Quote to buy: 25 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus Marginal Cost: 25 [\$/MWh] Power: 900 [MW] TCC payment: 12.6 [k\$/h] Payment: 22.5 [k\$/h] Profit: 9 [k\$/h]
Load 3 - Fixed	Power: 900 [MW]	Power: 900 [MW]

With or without the line limit, the participants quote very different values than their true cost or true willingness to pay. Due to these distortions in the quotes, the resulting

allocation of power is not efficient.

#### Case 4 - Load Company Owns a Transmission Congestion Contract for Transmission from 2 to 3

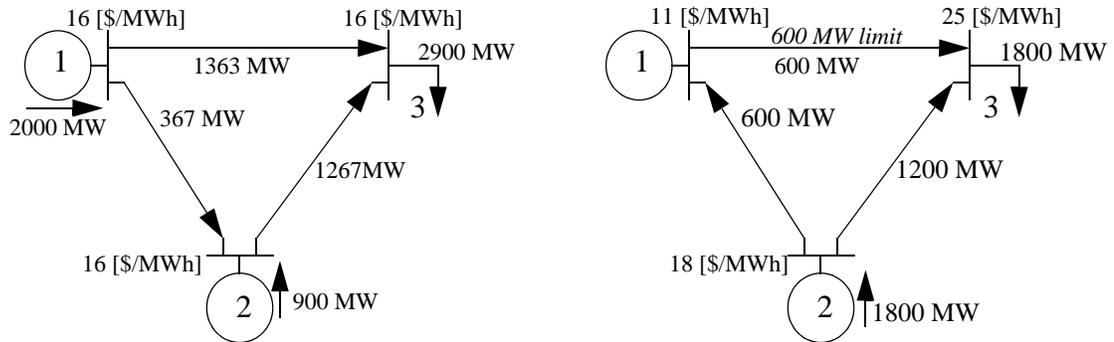
In this case load company 3 owns the transmission congestion contract for transmission from 2 to 3. In the case with no transmission limits the quotes converge to the quotes 8, 16 and 20 [\$/MWh] for generator 1, generator 2 and load 3, respectively. On the other hand, the case with a transmission limit on line 1-3, the quotes converge to 13, 18 and 25 [\$/MWh] for generator 1, generator 2 and load 3, respectively. These results are the same as the case in which the load company 3 had a transmission congestion contract for transmission from 1 to 3, but since both contracts include the congested line (1-3) the results are equivalent. The result from the simulation is displayed in the following figure.



**Figure 8-12: Transmission Congestion Contract (Load Company 3) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

Notice that applying a transmission congestion contract scheme with no congestion produces the same results than the bus marginal cost scheme (see Figure 8-5), this is due to the fact that the contract will be useful to generator company 2 only when congestion arises (making marginal prices differ from one bus to another). In the case with the transmission limit, load company 3 is willing to quote 25 [\$/MWh] (4 [\$/MWh] more than its true cost 21 [\$/MWh]) with the objective of causing congestion, and with that, obtaining more profit of its transmission congestion contract payment.

The generations, loads, and the line flows for both situations are shown in the following figure.



**Figure 8-13: Equilibrium for Transmission Congestion Contract (Load Company 3) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

The contract in this case will pay 12.6 [k\$/h] to the load company due to the difference in marginal cost between buses 2 and 3 (1800 MW times 25 - 18 = 7 [\$/MWh]). This amount is more than enough to cover the losses that this company will incur for

increasing its willingness to pay above its true value, and still give the company a profit of 9 [k\$/h]. All the details of the equilibrium are include in the following table (profits, marginal costs, payments, TCC payments, etc.)

**Table 8-7: Equilibrium for Transmission Congestion Contract (Load Company 3) with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

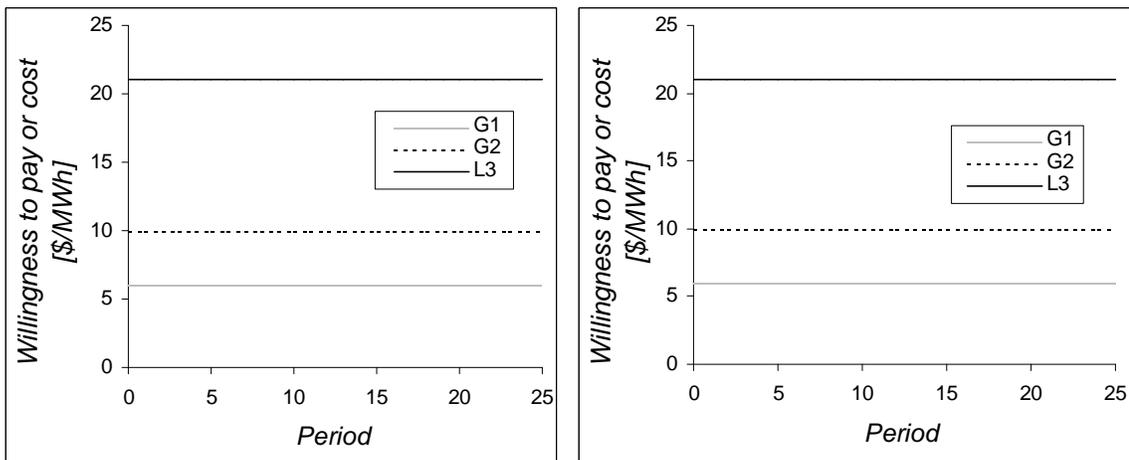
	No transmission limits	600-MW limit on line 1-3
Generator 1	Quote to sell: 8 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 2000 [MW] Revenue received: 32 [k\$/h] Profit: 20 [k\$/h]	Quote to sell: 13 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 11 [\$/MWh] Power: 0 [MW] Revenue received: 0 [k\$/h] Profit: 0 [k\$/h]
Generator 2	Quote to sell: 16 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 16 [\$/MWh] Power: 900 [MW] Revenue received: 14.4 [k\$/h] Profit: 5.4 [k\$/h]	Quote to sell: 18 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 18 [\$/MWh] Power: 1800 [MW] Revenue received: 32.4 [k\$/h] Profit: 14.4 [k\$/h]
Load 3 - Variable	Quote to buy: 20 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus Marginal Cost: 16 [\$/MWh] Power: 2000 [MW] TCC payment: 0 [k\$/h] Payment: 32 [k\$/h] Profit: 10 [k\$/h]	Quote to buy: 25 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus Marginal Cost: 25 [\$/MWh] Power: 900 [MW] TCC payment: 12.6 [k\$/h] Payment: 22.5 [k\$/h] Profit: 9 [k\$/h]
Load 3 - Fixed	Power: 900 [MW]	Power: 900 [MW]

With or without the line limit, the participants quote very different values than their true cost or true willingness to pay. Due to these distortions in the quotes, the resulting

allocation of power is not efficient.

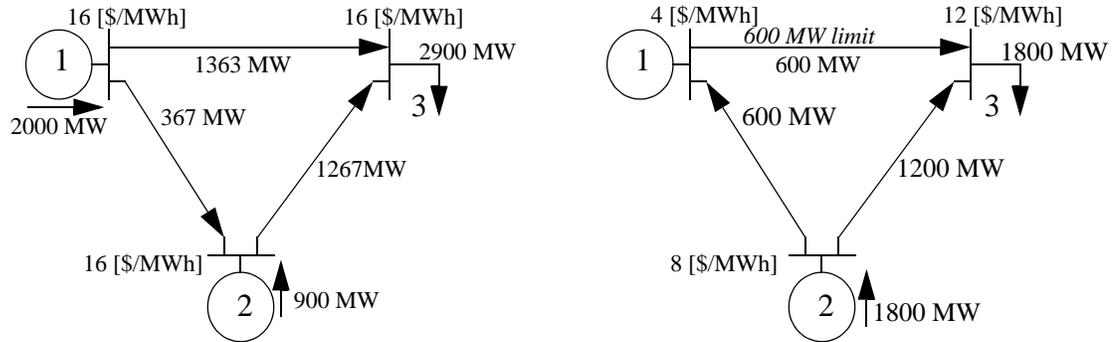
### 8.2.4 Incentive Compatible Payment Scheme

In this case the companies receive a payment covering the cost (or willingness to pay) they quote to sell (or buy) plus an information payment (for details see Chapter 5). For both the cases with and without transmission limits the quotes converge to the quotes 6, 10 and 21 [\$/MWh] for generator 1, generator 2 and load 3, respectively. These quotes correspond exactly to the true cost of the generator companies and the true willingness to pay of the load.



**Figure 8-14: Incentive Compatible Mechanism with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

Notice that quotes start and finish at the values of the true marginal cost and true willingness to pay. The generations, loads, and the line flows for both situations are shown in the following figure.



**Figure 8-15: Equilibrium for the Incentive Compatible Mechanism with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

All the details of the equilibrium are included in the following table (profits, marginal costs, payments, etc.).

**Table 8-8: Equilibrium for the Incentive Compatible Mechanism with no Transmission Limit (left), and with a 600-MW Limit on Line 1-3 (right)**

	No transmission limits	600-MW limit on line 1-3
Generator 1	Quote to sell: 6 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: 10 [\$/MWh] Power: 2000 [MW] Info. Compensation: 12.73 [k\$/h] Revenue received: 24.73 [k\$/h] Profit: 12.73 [k\$/h]	Quote to sell: 6 [\$/MWh] True cost: 6 [\$/MWh] Bus marginal cost: -1 [\$/MWh] Power: 0 [MW] Info. Compensation: 0 [k\$/h] Revenue received: 0 [k\$/h] Profit: 0 [k\$/h]
Generator 2	Quote to sell: 10 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 10 [\$/MWh] Power: 900 [MW] Info. Compensation: 12.73 [k\$/h] Revenue received: 21.73 [k\$/h] Profit: 12.73 [k\$/h]	Quote to sell: 10 [\$/MWh] True cost: 10 [\$/MWh] Bus marginal cost: 10 [\$/MWh] Power: 1800 [MW] Info. Compensation: 3.28 [k\$/h] Revenue received: 21.38 [k\$/h] Profit: 21.38 [k\$/h]
Load 3 - Variable	Quote to buy: 21 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus marginal cost: 10 [\$/MWh] Power: 2000 [MW] Info. Compensation: 12.73 [k\$/h] Payment: 42 [k\$/h] Profit: 30 [k\$/h]	Quote to buy: 21 [\$/MWh] Willingness to pay: 21 [\$/MWh] Bus marginal cost: 21 [\$/MWh] Power: 900 [MW] Info. Compensation: 3.7 [k\$/h] Payment: 18.9 [k\$/h] Profit: 3.7 [k\$/h]
Load 3 - Fixed	Power: 900 [MW]	Power: 900 [MW]

### 8.3 Summary

The efficiency of a payment scheme depends on how far the quotes differ from the participants true costs (generators) and true willingness to pay (loads). In the case of

participant distorting their true values, efficiency cannot be expected. In the following table a summary can be seen with the quotes for all the cases.

**Table 8-9: Quotes for All the Participants for all the Simulations**

Payment Scheme	No transmission limits. quotes for generator 1, generator 2 and load 3	600-MW limit on line 1-3, quotes for generator 1, generator 2 and load 3
True cost (generators) and willingness to pay (loads)	6, 10 and 21 [\$/MWh]	6, 10 and 21 [\$/MWh]
Traditional	11, 12 and 12 [\$/MWh]	9, 11 and 14 [\$/MWh]
Bus marginal Cost	8, 16 and 20 [\$/MWh]	9, 11 and 14 [\$/MWh]
Transmission Congestion Contract, case 1 - generator 1 owns a TCC for 1-3	8, 16 and 20 [\$/MWh]	1, 25 and 12 [\$/MWh]
Transmission Congestion Contract, case 2 - generator 2 owns a TCC for 2-3	8, 16 and 20 [\$/MWh]	8, 8 and 12 [\$/MWh]
Transmission Congestion Contract, case 3 - load 3 owns a TCC for 1-3	8, 16 and 20 [\$/MWh]	13, 18 and 25 [\$/MWh]
Transmission Congestion Contract, case 4 - load 3 owns a TCC for 2-3	8, 16 and 20 [\$/MWh]	13, 18 and 25 [\$/MWh]
Incentive Compatible	6, 10 and 21 [\$/MWh]	6, 10 and 21 [\$/MWh]

So from the table above it is clear the only algorithm that insures efficiency is the incentive compatible mechanism.

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## CHAPTER 9 CONCLUSIONS AND FUTURE CHALLENGES

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The electricity sector is changing. The way electric energy suppliers do business and the way in which they interact with each other is definitely changing. All this transformation is due to the creation of open markets in the electricity sector. Regulators have been successful in deregulating other network-based services/products. Nevertheless, the electricity market has proven to be a very difficult and unique environment, and the real benefits of deregulating the electricity market are yet to come.

The reasons why the deregulation of the electricity market has been difficult are:

- There are no rules to create an efficient market in electricity. There is no 100% satisfactory market organization scheme among the countries that have already gone through this process.
- Most of the people that are going to be part of the new deregulated industry are those that were working in the highly regulated environment (mostly utility employees and regulators). There are new things to learn, new concepts and ideas, and not everyone will catch up at the same pace.
- The electricity industry is one of the most investment intensive industries in the U.S. and it also serves almost every inhabitant. Therefore, no mistakes can

be allowed and precautions are going to be taken in order to permit a smooth transition to deregulation. Also, many parties have invested huge amounts of resources and they are going to bring pressure if they feel their interests are in danger. Regulators need to decide not only what is best, but also consider the impact on the customers and on the investors.

## **9.1 Conclusions**

The main conclusion of this thesis is the difficulty of organizing the electric power market using methodologies based on bus marginal cost payment. As was discussed in this thesis, the bus marginal costs depend on the cost function of the generators and on the willingness to pay of the consumer companies, and these companies are not going to willingly give up key information about their operations.

This thesis proposes the used of a new methodology, called the incentive compatible mechanism, that will correctly deal with congestion management, the weak point of the other methods. The advantages and disadvantages of the incentive compatible mechanism are contrasted with methods based on bus marginal cost using simulations.

## **9.2 Future Challenges**

In this section are presented some future challenges of this thesis. Readers are encourage to continue this line of research using any of the possible path proposed

below.

### **9.2.1 Simulation**

The simulation of an algorithm has shown to provide a much clearer idea about its advantages and disadvantages than just a plain description of the algorithm itself, and in this process, the use of a more realistic simulation framework can help to obtain more valuable conclusions. Among the characteristics of a market simulation that were implemented in this thesis are: multi-period simulations, variable load and multi-segment cost function. Some other characteristics that can be included in a simulation are: different agent algorithms, assessment of the security of the system, etc.

### **9.2.2 Full Alternating Current (AC) Model**

Most of the development in power system economics is done using a DC (Direct Current) model. Most researchers consider this approximation to be very good, but the extension of this development to a full alternating (AC) model appears as a critical continuation of this work.

### **9.2.3 Ancillary Services**

The power system objective is to bring active power to consumers, but to have the system up and running the operators also need some secondary products, called ancillary services (supply of reactive power, frequency regulation, supply of transmission losses, etc.). These products need to be priced so they can be assigned to agents who have to pay for them and agents who have to provide them. In general, the

most accepted idea is to include all the ancillary services in a secondary market (parallel to the main electricity market).

#### **9.2.4 Large Test Cases**

Most methodologies proposed to structure and operate a market will always work in small systems. But clearly these methods need to be shown as useful in managing a large and complicated system. One of the difficulties in setting up a large system simulation is the problem of gathering all the data that, if it is available. The simulation itself can also bring up problems such as excessive computing time or an unmanageable dimensionality. Nevertheless, there is extensive literature in technical journals and books in this area than can help researchers to overcome these and any other difficulties in simulating a large case.

#### **9.2.5 Real Implementation**

The transition from the simulation in a laboratory to an actual electricity marketplace brings up a new set of additional difficulties. The monitoring of the system in real time, communication between the agents and the power pool, communication among agents, settlement system among agents, enforcement of policies and implementations, etc. All these are important topics which are not considered in the laboratory simulations, but are essential in to implement a real electricity market.

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## APPENDIX A PAYMENT FUNCTION FOR CASES WITH LINEAR OBJECTIVE FUNCTIONS

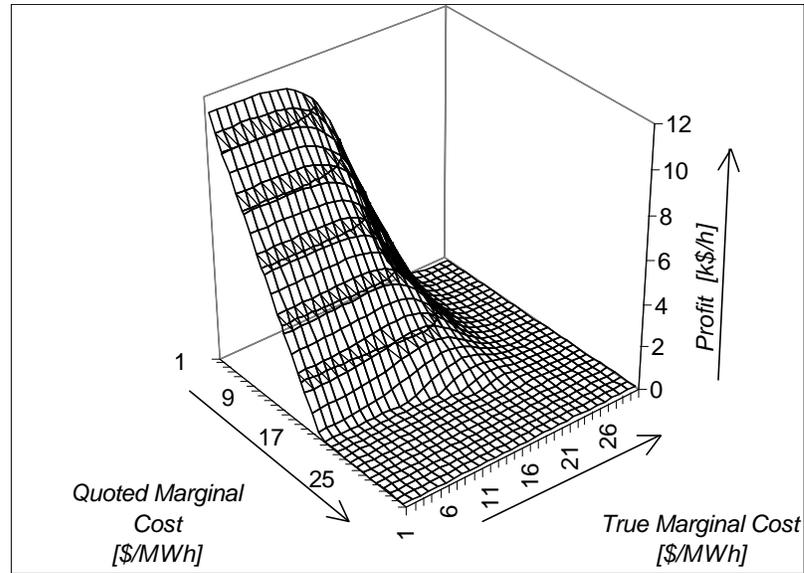
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This appendix presents an informal derivation of the payment function for the incentive compatible mechanism described in Chapter 5.

In this setting, a participant tries to maximize its expected profit, which is defined as the expected revenues minus the expected cost (in this demonstration the agent is a generator, the same conclusions can be obtained in the case of a consumer). The expected revenues are the sum of a cost payment,  $\hat{c}_i \cdot \bar{P}_i(\hat{c}_i)$  and an information payment,  $\bar{\tau}_i(\hat{c}_i)$ . On the other hand, the total costs are given by  $c_i \cdot \bar{P}_i(\hat{c}_i)$ . The expression for the expected profit function, depending on the true cost,  $c_i$ , and the quoted cost,  $\hat{c}_i$  would be:

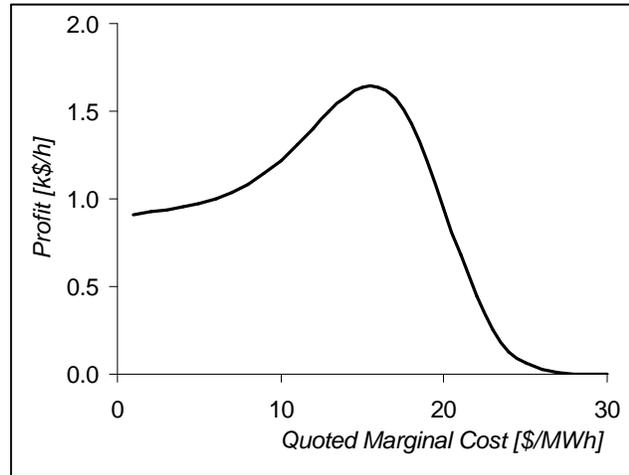
$$\pi_i(\hat{c}_i, c_i) = (\hat{c}_i - c_i) \cdot \bar{P}_i(\hat{c}_i) + \bar{\tau}_i(\hat{c}_i) \quad (\text{A.1})$$

The curve shown in Figure A-1 is an example of the function  $\pi_i(\hat{c}_i, c_i)$  in a two-dimensional domain.



**Figure A-1: Profit Function**

To achieve incentive compatibility, the profit function should not give any incentive to generator  $i$  to deviate its quote from its true cost. For example, if the profit function is fixed at any feasible true cost (e.g., 16 [\$/MWh]) the resulting curve is maximized at a marginal cost provided equal to true cost level (16 [\$/MWh] as shown in Figure A-2).



**Figure A-2: Profit vs. Quoted Marginal Cost**

A function  $s(\cdot)$  can be defined as the optimal marginal cost to quote given a true cost:

$$\hat{c}_i = s(c_i) \quad (\text{A.2})$$

So eq. (A.1) can be rewritten as,

$$\pi_i(s(c_i), c_i) = (s(c_i) - c_i) \cdot \bar{P}_i(s(c_i)) + \bar{\tau}_i(s(c_i)) \quad (\text{A.3})$$

If the first derivative of  $\pi_i(s(c_i), c_i)$  is taken with respect to  $c_i$  it can be obtained that,

$$\frac{d}{dc_i} \pi_i(s(c_i), c_i) = D_1 \pi_i(s(c_i), c_i) \cdot \frac{d}{dc_i} s(c_i) + D_2 \pi_i(s(c_i), c_i) \quad (\text{A.4})$$

By the first order necessary condition of the maximization in  $\hat{c}_i$ , the first derivative of

$\pi_i(s(c_i), c_i)$  with respect to  $\hat{c}_i = s(c_i)$  is zero. Therefore the above expression can be reduced to:

$$\frac{d}{dc_i} \pi_i(s(c_i), c_i) = D_2 \pi_i(s(c_i), c_i) = -\bar{P}_i(s(c_i)) \quad (\text{A.5})$$

The incentive compatibility property implies that for any true cost,  $c_i$ , the optimal quoted cost equals the true value,

$$\hat{c}_i^* = s(c_i) = c_i \quad (\text{A.6})$$

so eq. (A.5) would be,

$$\frac{d}{dc_i} \pi_i(c_i, c_i) = -\bar{P}_i(c_i) \quad (\text{A.7})$$

Applying the integral operator between the upper bound of the probability distribution,  $\bar{c}_i$ , and a generic  $c_i$ , and then,

$$\int_{\bar{c}_i}^{c_i} \frac{d}{dc_i'} \pi_i(c_i', c_i') dc_i' = \pi_i(c_i, c_i) - \pi_i(\bar{c}_i, \bar{c}_i) = \int_{c_i}^{\bar{c}_i} \bar{P}_i(c_i') dc_i' \quad (\text{A.8})$$

Replacing the value of  $\pi_i$  from eq. (A.1) it can be obtained,

$$(c_i - c_i) \cdot \bar{P}_i(c_i) + \tau_i(c_i) - (\bar{c}_i - \bar{c}_i) \cdot \bar{P}_i(\bar{c}_i) - \tau_i(\bar{c}_i) = \int_{c_i}^{\bar{c}_i} \bar{P}_i(c_i') dc_i' \quad (\text{A.9})$$

cancelling out the zero terms,

$$\tau_i(c_i) - \tau_i(\bar{c}_i) = \int_{c_i}^{\bar{c}_i} \bar{P}_i(c_i') dc_i' \quad (\text{A.10})$$

Finally, the generic function  $\tau_i(c_i)$  that solves the above is,

$$\tau_i(c_i) = \int_{c_i}^{\bar{c}_i} \bar{P}_i(c_i') dc_i' + K \quad (\text{A.11})$$

In this application one can set K to zero, but any real number can be used if it does allow the individual rationality property to hold (some negative numbers will not do it).

In order to prove incentive compatibility, one can subtract the profit function  $\pi_i$  for the real cost from the profit for any given cost. If the result is non-positive, generator i will never get a positive gain if it deviates from its true marginal cost.

$$\pi_i(\hat{c}_i, c_i) - \pi_i(c_i, c_i) = (\hat{c}_i - c_i) \bar{P}_i(\hat{c}_i) + \int_{\hat{c}_i}^{\bar{c}_i} \bar{P}_i(c_i') dc_i' - \int_{c_i}^{\bar{c}_i} \bar{P}_i(c_i') dc_i' \quad (\text{A.12})$$

Regrouping,

$$= (\hat{c}_i - c_i) \bar{P}_i(\hat{c}_i) + \int_{\hat{c}_i}^{c_i} \bar{P}_i(c_i') dc_i' \quad (\text{A.13})$$

Then,

$$\pi_i(\hat{c}_i, c_i) - \pi_i(c_i, c_i) = (\hat{c}_i - c_i)(\bar{P}_i(\hat{c}_i) - \bar{P}_i(\xi)) \quad (\text{A.14})$$

for some value  $\xi$  strictly between  $c_i$  and  $\hat{c}_i$  (mean-value theorem of integration).

Considering that  $\bar{P}_i(\hat{c}_i)$  is a decreasing function two cases need to be analyzed in order to figure out the sign of the equation above.

- If  $c_i$  is greater than  $\hat{c}_i$  then  $(\hat{c}_i - c_i)$  is negative and  $\bar{P}_i(\hat{c}_i) - \bar{P}_i(\xi)$  is positive because  $\xi$  is greater than  $\hat{c}_i$  (and  $\bar{P}_i(\hat{c}_i)$  is decreasing). Therefore the whole expression is negative.

- If  $\hat{c}_i$  is greater than  $c_i$  then  $(\hat{c}_i - c_i)$  is positive and  $\bar{P}_i(\hat{c}_i) - \bar{P}_i(\xi)$  is negative because  $\xi$  is greater than  $\hat{c}_i$  (and  $\bar{P}_i(\hat{c}_i)$  is decreasing). Therefore the whole expression is negative. Therefore  $\pi_i(c_i, c_i)$  is always greater or equal than  $\pi_i(\hat{c}_i, c_i)$ , and the incentive compatibility property holds.

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## APPENDIX B PAYMENT FUNCTION FOR MULTI-SEGMENT OBJECTIVE FUNCTIONS

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This appendix presents an informal derivation of the incentive compatible mechanism including companies whose cost curve can be defined with a piece-linear curve. (In the case of a generator company this means either a company owning units in more than one bus or generators defined with more than one marginal cost segment.)

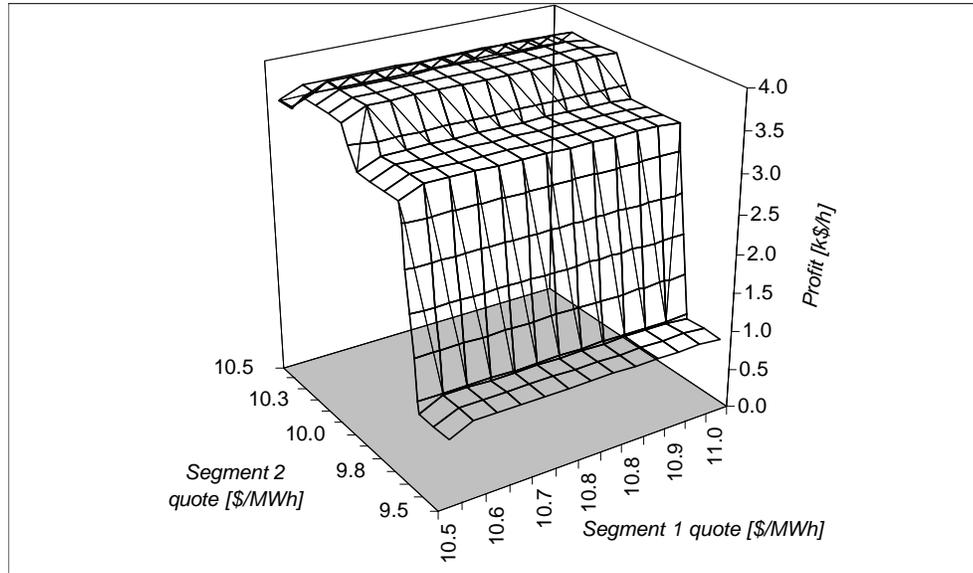
In this setting, a generator tries to maximize its expected profit, which is defined as the expected revenues minus the expected cost (this demonstration assumes that the agent is a generator, the same conclusions can be obtained in the case of a consumer). The expected revenue is the sum of a cost payment,  $\hat{c} \cdot \bar{P}(\hat{c})$  and an information payment,  $\tau(\hat{c})$ . On the other hand, the total costs are given by  $c \cdot \bar{P}(\hat{c})$ , where  $\bar{P}(\hat{c})$  is the expected production for a given quote vector,  $\hat{c}$ . The expression for the expected profit function, depending on the true cost vector,  $c$ , and the quote cost vector,  $\hat{c}$  would be:

$$\pi(\hat{c}, c) = (\hat{c} - c) \cdot \bar{P}(\hat{c}) + \tau(\hat{c}) \quad (\text{B.1})$$

If there are  $n$  different generator segments, the function  $\pi(\hat{c}, c)$  is defined in the  $2n$ -dimensional domain.

To achieve incentive compatibility, the profit function should not give any incentive to generator to deviate its claim from its true cost. For example, fixing the profit function

at any feasible true marginal cost (e.g., 16 [\$/MWh]) the resulting curve is maximized at a marginal cost provided equal to true cost level (16 [\$/MWh] as shown in Figure B-1).



**Figure B-1: Profit Function**

Defining the function  $s(\cdot)$  as the optimal marginal cost to claim given a true cost:

$$\hat{c} = s(c) \tag{B.2}$$

So eq. (B.1) can be rewritten as,

$$\pi(s(c), c) = (s(c) - c) \cdot \bar{P}(s(c)) + \tau(s(c)) \tag{B.3}$$

Taking the first derivative of  $\pi(s(c), c)$  with respect to  $c_i$ ,

$$\frac{d}{dc_i}\pi(s(c), c) = \sum_{j=1}^n \frac{d}{ds_j(c)}\pi(s(c), c)\frac{d}{dc_i}s_j(c) + \frac{d}{dc_i}\pi(s(c), c) \quad (\text{B.4})$$

By the first order necessary condition of the maximization in  $\hat{c}_i$ , the first derivative of  $\pi(s(c), c)$  with respect to all of the terms  $\hat{c}_j = s_j(c)$  are zero (for j equal 1 to n).

Therefore the above expression can be reduced to:

$$\frac{d}{dc_i}\pi(s(c), c) = \frac{\partial}{\partial c_i}\pi(s(c), c) = -\bar{P}_i(s(c)) \quad (\text{B.5})$$

The incentive compatibility property implies that for any true cost,  $c_i$ , the optimal quote equals the true value,

$$\hat{c}^* = s(c) = c \quad (\text{B.6})$$

so eq. (B.5) would be,

$$\frac{d}{dc_i}\pi(c, c) = -\bar{P}_i(c) \quad (\text{B.7})$$

Applying the integral operator between the upper bound of the probability distribution,

$\bar{c}$ , and a generic  $c$ ,

$$\int_{\bar{c}}^c \frac{d}{dc'} \pi(c', c') \cdot dc' = \pi(c, c) - \pi(\bar{c}, \bar{c}) = \int_c^{\bar{c}} \bar{P}(c') \cdot dc' \quad (\text{B.8})$$

Replacing the value of  $\pi$  from eq. (B.1),

$$(c - c) \cdot \bar{P}(c) + \tau(c) - (\bar{c} - \bar{c}) \cdot \bar{P}(\bar{c}) - \tau(\bar{c}) = \int_c^{\bar{c}} \bar{P}(c') \cdot dc' \quad (\text{B.9})$$

cancelling out the zero terms,

$$\tau(c) - \tau(\bar{c}) = \int_c^{\bar{c}} \bar{P}(c') \cdot dc' \quad (\text{B.10})$$

Finally, the generic function  $\tau(c)$  that solves the above is,

$$\tau(c) = \int_c^{\bar{c}} \bar{P}(c') \cdot dc' + K \quad (\text{B.11})$$

In this application one can set  $K$  to zero, but any real number can be used if it does allow the individual rationality property to hold (some negative numbers will not do it).

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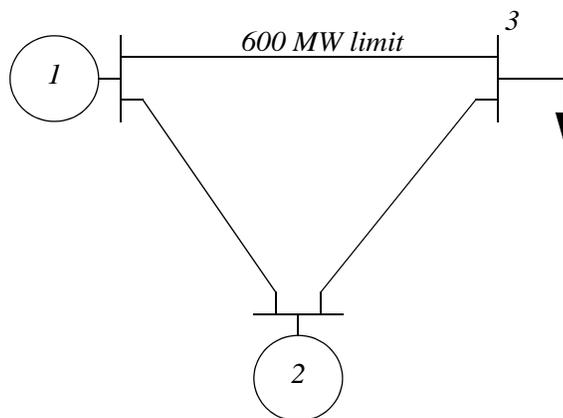
## APPENDIX C ECONOMIC DISPATCH DATA

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### C.1 3-bus System

Even though the smallest case that includes congestion has two buses, the most common system in the literature to show the effects of congestion is a 3-bus-test case as shown in Figure C-1. Also it is commonly include a restriction of 600 MW in transmission line 1-3.

The data for this system is included in the following tables.



**Figure C-1: 3-bus System**

**Table C-1: Agent Data for the 3-bus System**

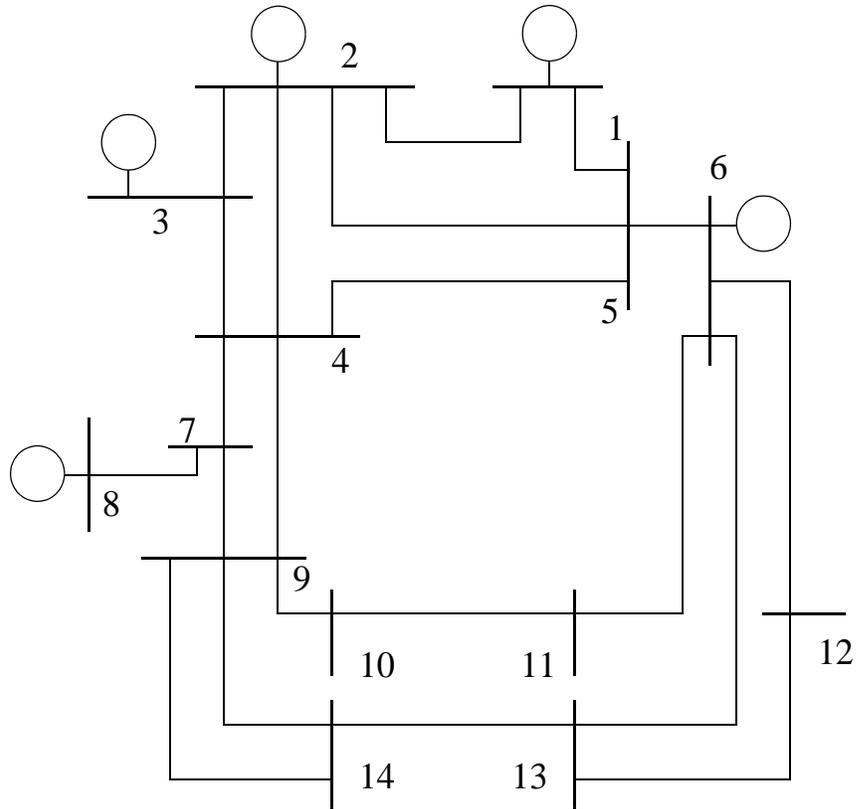
Agent	Type	Location (bus)	Minimum Power [MW]	Maximum Power [MW]
1	Generator	1	0	2000
2	Generator	2	0	2000
3	Load	3	0	2000

**Table C-2: Network Data for the 3-bus System**

From	To	Resistance [p.u.]	Reactance [p.u.]	Capacity [MW]
1	2	0.2	0.03	not limited
1	3	0.2	0.03	6000
2	3	0.2	0.03	not limited

## C.2 IEEE-14-bus System

One of the most widely used test cases in power system is the IEEE-14-bus system. In this application a slightly modified version was used. The data for this system is included in the following tables.



**Figure C-2: IEEE-14-bus Case**

The agent data and the network data is included in the following two tables.

**Table C-3: Agent Data for the IEEE-14-bus Case**

Agent	Type	Location (bus)	Minimum Power [MW]	Maximum Power [MW]
1	Generator	1	0	200
2	Generator	2	0	800
3	Generator	3	0	800
4	Generator	6	0	800
5	Generator	8	0	800

**Table C-4: Network Data for the IEEE-14-bus Case**

From	To	Resistance [p.u.]	Reactance [p.u.]	Capacity [MW]
1	2	0.019	0.06	200
1	5	0.54	0.22	50
2	3	0.047	0.19	160
2	4	0.058	0.18	145
2	5	0.057	0.17	130
3	4	0.067	0.17	125
4	5	0.013	0.04	60
4	7	0.000	0.51	50
4	9	0.000	0.56	30
5	6	0.000	0.25	10
6	11	0.095	0.19	50
6	12	0.1229	0.25	30
6	13	0.066	0.13	60
7	8	0.000	0.18	80
7	9	0.000	0.11	30
9	10	0.0318	0.08	30

**Table C-4: Network Data for the IEEE-14-bus Case**

From	To	Resistance [p.u.]	Reactance [p.u.]	Capacity [MW]
9	14	0.1271	0.27	10
9	14	0.1271	0.27	10
10	11	0.082	0.19	40
12	13	0.221	0.20	10
13	14	0.171	0.35	35