Consumers coordination and cooperation in transmission cost allocation

Juan M. Zolezzi, Member, IEEE, and Hugh Rudnick, Fellow, IEEE

Abstract—The profound change in the electric industry worldwide in the last twenty years assigns an increasing importance to electric market agents’ interaction, even if these are competitive markets like generation and commercialization, or non competitive transmission and distribution markets. The agent’s cooperation and coordination through coalition formation in cost allocation of investment, electric network operation and maintenance, arises as an attractive solution, if one has an appropriate technical and economic modeling. The obtained solutions in such cases are efficient, fair and equitable to participant agents. A transmission cost allocation method is presented, based on cooperative game theory and transmission network capacity use by consumer agents. It is applied to the main Chilean interconnected system and the obtained results are compared with traditional methodologies.

Index Terms—Coalition Formation, Cooperative Game Theory, Nucleolus, Open Access, Transmission Cost Allocation, Transmission Expansion, Shapley Value.

I. INTRODUCTION

The reformed electric industry scheme sets the transmission sector at the center of the electric market [1], where an open non discriminative access to the network is made available to competitive generators and consumers.

The scope and scale economies present in transmission, make this sector a natural monopoly that requires regulation in order to reach efficiency investment and operation [2]. It also requires to define a price and payment system that doesn’t distort the investment decisions in new generation, generator’s operation decisions and consumer’s demand decisions [3].

The usual marginal pricing scheme in these markets, based on marginal costs or bids, is inadequate to recover the transmission total costs; marginal costs are lower than average costs, requiring a complementary payment or charge to allow recovering the difference [4]. The way to distribute the complementary charge among transmission system users has been a long debated subject among agents. Each country has found its own solution [5], according with its transmission system reality.

Methods based on physical use by agents arise among complementary charge allocation schemes [6]. The difficulty of this “network use” scheme is in identifying exactly the transmission installations that are used by each system agent. This is due to the fact that the flow distributes through the network according to physical laws which do not keep relation with supply contracts established by producers and consumers. The beneficiaries’ method is another alternative that has been proposed, with solid economic bases, but it faces difficulties in practical applications [7].

Cooperative game theory arises as a most convenient tool to solve cost allocation problems. The solution mechanisms of cooperative game theory behave well in terms of fairness, efficiency and stability, qualities required for the correct allocation of transmission costs [8]. Nevertheless, proposals to date are still in a developing stage; contributions have been formulated in wheeling transactions [9], and in the allocation of expansion costs [10]- [11].

This paper presents the application of a method to allocate charges among users of a transmission system. The method is based in a model that integrates cooperation and coordination among the agents as basic principles. It also considers the physical and economic use of the network by the different agents. It assumes a rational behavior of the agents, the formation of coalitions among agents, along with cooperative game solution mechanisms.

The proposed method [12] is simple to apply and it has an adequate physical and economic understanding of the electricity transmission problem. It provides adequate signals to agents making investment decisions, ensuring fairness, efficiency and stability in the resultant allocation. The method is applied for cost allocation to consumers in a network.

II. TRADITIONAL CONSUMERS COST ALLOCATION

Two alternative methods aiming at determining the contribution of each agent to the flow of each line are compared, with the first one using the superposition principle while the second uses the proportionality principle. Main characteristics of each method are presented.

A. Rudnick’s method

It was the first attempt to relate transmission pricing to system use [4], utilizing distribution factors previously defined for the study of power system security. The method pretends to obtain parameters that indicate the level of utilization of the network by generators and loads.
There are three types of distribution factors. The A or GSDF distribution factors (called generalized shift distribution factors) measure the impact over the flow in a given line considering incremental changes in power injection or withdrawal at all buses, except the reference bus. The GGDF distribution factors (generalized generation distribution factors) measure the total impact (not incremental) of power injection by generators over the flow in a given line. Finally, the GLDF distribution factors (generalized load distribution factors) measure the total impact of negative injections, which correspond to loads, over the flow in a certain line.

1) A or GSDF factors

A \( A_{i-k,b} \) factor, defined by means of a sensitivity analysis, relates a variation in the injected power \( \Delta P_{I_b} \) in a bus \( b \) to a variation \( \Delta F_{i-k} \) in the flow through line \( i-k \). It considers that, while injections from generators and loads in other buses, as well as losses, are maintained constant.

\[
A_{i-k,b} = \frac{\Delta F_{i-k}}{\Delta P_{I_b}}
\]

Therefore, for a variation in all the injections, the variation in the flow in a particular line will be given by:

\[
\Delta F_{i-k} = \sum_{b \in R} A_{i-k,b} \cdot \Delta P_{I_b}
\]

a change in the injected power in any bus is absorbed by a similar negative change \( \Delta P_{I_R} \) in the reference bus \( R \)

\[
\sum_{b \in R} \Delta P_{I_b} + \Delta P_{I_R} = 0
\]

The A factors are obtained from a DC power flow, which only models reactances, and are calculated as follow:

\[
A_{i-k,b} = \frac{X_{i-b} - X_{k-b}}{X_{i,k}}
\]

where \( X_{i,b} \) and \( X_{k,b} \) are elements of the reactance matrix and \( X_{i,k} \) is the reactance of line between buses \( i \) and \( k \). These factors are independent of the operational conditions of the system (generation and load distribution), but depend on the grid configuration and the reference bus chosen.

2) L or GLDF factors

A \( L_{i-k,g} \) factor relates the total load from a consumer \( L_g \) in a bus \( g \), with the total flow \( F_{i-k} \) over a line \( i-k \). These factors emerge from the following equations:

\[
F_{i-k} = \sum_{g} C_{i-k,g} \cdot L_g
\]

These factors are independent of the election of the reference bus, but depend on the system configuration and on its operational conditions. GLDF factors are obtained from A factors:

\[
C_{i-k,g} = A_{i-k,g} - C_{i-k,R}
\]

where \( L_{i-k,R} \) is defined as:

\[
C_{i-k,R} = \left( \sum_{p \in R} A_{i-k,p} \cdot L_{p} \right) / \sum_{g} L_g
\]

If some L factors turn out to be negative, they are considered nil for the allocation. Thus, to determine the level of contribution of a consumer \( b \) to the flow \( F_{i-k} \) of a particular line \( i-k \), equation (7) can be used, where \( C_{i-k,b} \) would be equal to \( C_{i-k,b} \) if the factor has the same sign as the flow and would be nil if the factor has an opposite sign.

\[
FP_{i-k,b} = \frac{\left( C_{i-k,b} \cdot L_b \right)}{\sum_{g} C_{i-k,g} \cdot L_g}
\]

B. Bialek’s method

This average participation method [16] aims at tracing the flows of electricity through power networks. It allows quantifying how much of the active or reactive power flows from a particular source to a specific load. It also allows quantifying the contribution from each generator or load to flows and losses in a given line.

It uses a proportionality principle, which states that for any bus there are lines that inject power and others that evacuate power. The proposed algorithm uses a non-loss power flow as a base. However, the author presents different alternatives to allocate the losses in the lines.

If the problem is analyzed from the perspective of consumers, the power withdrawal in each bus of the system are given by:

\[
P_i = \sum_{l:i \in l} P_{i-l} + P_{i,i} = \sum_{l:i \in l} c_{il} \cdot P_l + P_{i,i} \quad \forall i = 1,2,\ldots,n
\]

where \( P_i \) is the total flow through bus \( i \), \( a_{i}^{(l)} \) is the set of buses fed directly from the node \( i \) (the flow must go from bus \( i \) to other buses), \( P_{i,i} \) is the consumption in bus \( i \) and \( P_{i,l} \) is the flow in line \( l-i \), where

\[
c_{il} = \frac{P_{i,l}}{P_i}
\]

and arranging (9):

\[
P_i = \sum_{l:i \in l} c_{il} \cdot P_l + P_{i,i} \quad \forall A_{l} P = P_{l}
\]

where \( A_{l} \) is an (nxn) distribution matrix per withdrawal powers, \( P \) is the vector of bus flows and \( P_{l} \) is the vector of loads bus.

The elements of matrix \( A_{l} \) are defined as follow:
The proposed method analyses each transmission line at a particular transmission line is determined by playing a cooperative game. The participation of an agent in covering the cost of a line is determined by playing a cooperative game. The method considers the maximum line flow conditions as a measure of line use. Thus, the characteristic function of the cooperative game for each line is based on the stand-alone requirement for each consumer agent or possible coalition of agents.

Each player or agent evaluates its transmission requirement for a given system operation condition, either acting separately or cooperating in a coalition with other agents also requiring the transmission network. To perform this evaluation, each agent must assess its use of each transmission line for a maximum flow condition. The game characteristic function is defined by the stand-alone requirement \( f_{ik} \) for each agent and for each potential coalition \( S \). Thus, the game characteristic function \( C_{ik}^S \), for line \( k \) and each potential coalition \( S \), will be:

\[
C_{ik}^S = f_{ik}^S \quad \forall S \subseteq NA
\]

The usefulness for each agent or agent coalition of utilizing line \( k \) is implicit in (15), through the line flow requirement. Thus, to represent the usefulness of the line it is possible to associate to each MW flow a monetary unit, without producing distortions in the game solution [13]. It is equivalent to solve the game considering the MW usefulness for each agent or agent coalitions.

There are no restrictions to the formation of coalitions among the agents; they are intelligent and rational, i.e. they create coalitions, which minimize their cost participation for given transmission lines. The sub-additive cost condition allows the coalitions to be formed, and further that all market agents will participate, forming the “grand coalition”.

The power flow for line \( k \) from bus \( l \) to bus \( m \) for coalition \( S \) is calculated using a DC model, as:

\[
f_{ik}^S = \frac{1}{x_{i,m}^S} (\Theta_{i}^S - \Theta_{m}^S) \quad \forall S \subseteq NA
\]

and with phase angles \( \Theta \) given by

\[
\theta_i = B^{-1} P^i
\]

where \( P^i \) is the vector of injected powers (generation minus load) for each coalition \( S \). To determine the injected generations, an economic dispatch is performed, taking into account the generation variable costs and generation production level.

For all potential coalitions \( S \), the economic dispatch will determine line flows and these will determine the game characteristic function value for that coalition. The solution of the game may be obtained using any cooperative game solution mechanism, such as Nucleolus, Shapley Value, Kernel and other [13]. As a result, the payoff configuration PC (payoff is the word for cost allocation in game theory) for line \( k \) is obtained.

\[
PC_k = (x_k^S; \delta^k)
\]

where \( x_k^S \) is the payment vector and \( \delta^k \) is the coalition configuration for line \( k \) (see appendix). This payoff configuration allows to determine the percentage cost allocation of line \( k \) to agent \( i \). This payoff configuration allows to determine the percentage cost allocation CA of line \( k \) to agent \( i \).

\[
CA_{i,k} = \frac{x_i^k (+)}{\sum_{i=1}^{N} x_i^k (+)}
\]

where \( x_i^k (+) \) are the positive values of cost allocation.

The method does not consider compensation for negative values; these only reduce the magnitude of the positive
assignments.

As indicated before, a DC flow algorithm is used. For the cooperative game solutions, the Mathematica software is employed.

To determine the allocation of the total cost of a transmission system to each agent, it is necessary to add its allocations for each line, i.e.:

\[ C_i = \sum_{k=1}^{NL} C_{i,k} \cdot CL_{k} \]  

(20)

\[ C_i \]: total cost allocation for the transmission system, corresponding to agent \( i \).

\( CL_{k} \): real total cost of line in monetary units

\( NL \): number of transmission lines.

B. Game rules

Agents must cooperate and coordinate to share the costs of the transmission network that interconnects them. That network is assumed to be the property of a third party, interested in recovering investments, operation and maintenance costs. The network is assumed to have been adequately planned, in terms of transmission capacity, quality, security and backup. No congestion is assumed.

Given that there is open access; consumers may establish commercial contracts with any generators, without restrictions, other than generator capacities.

Agents participating in the market are intelligent, independent and rational, that is, they prefer to reduce their costs. Thus, they will be eager to cooperate in a rational economic environment. This means that no agent or coalition of agents will have a cost greater than its stand-alone cost, and the resultant game cost allocation will cover all transmission costs.

The method does not consider compensation for negative values; these only reduce the magnitude of the positive assignments.

IV. SIMULATIONS

The described methodology is applied to the Chilean Interconnected System (SIC) in a simplified 8 bus model (Fig 1). The SIC extends 1.740 Km. covering a territory of 323,250 km² equivalent to 42.3% of the country where 93% of the population lives. At the end of 2002, the SIC has 6,733 MW of installed power, with 39.9% thermal and 60.1% hydraulic, the maximum demand reached the 4,878 MW, while the gross generation of energy was located around the 31,971,3 GWh. Chilean SIC includes 14,019 circuit-kilometers of transmission lines and a transformation capacity of 23,795 MVA. SIC line data is given in Table I.

Comparative studies have been made with generalized load distribution parameters GLDF and average participations [15]. Additionally, in this case the studies were supplemented with the application of the COALA-IDEAS software, which allowed to model a negotiation process and communication among consumers, by means of two mechanisms designed for such an effect; Bilateral Shapley Value (BSV) and the Kernel Coalition Formation (KERNEL) [15].

When applying the proposed methodology, the operational conditions indicated in Table II were considered. The results of consumers participation (expressed in percentage of each agent's participation in the cost of the SIC transmission system) are presented in Table III and Figure 2. Also one can observe, in Table IV and Figure 3, the L7 consumer's responsibility in the different lines of the SIC; consumer L7 is a sunk consumer in bus 7. Table V and Figure 4 also show the results of L4 consumer's responsibility in the costs of the different lines of the SIC. One can observe that consumer L4 is the biggest consumer in the SIC.

![Fig. 1. 8 Bus SIC system.](image-url)
V. NUMERICAL COMPARISON

The responsibility of each consumer’s agent, in the transmission system cost and in their financing is recognized by the proposed methodology (Table III and Figure 2), indistinctly of the cooperative games resolution mechanism used (Shapley Value or Kernel). The GLDF method recognizes it in a similar form. The average participation method has a slightly different behavior in the agent’s responsibilities allocation.

In appendix B an example of the characteristic function of the game is shown, in this case for line 5-7.

In Table IV and Figure 3 it is possible to notice that, when there are sunk consumptions as L7 on bus 7, the method proposed by the two mechanisms of cooperative game resolution (Shapley Value and Kernel) reflect this consumption responsibility on line 1-2 and line 7-8, in the same way as GLDF methodology does. On the other hand, the average participation method does not recognize that responsibility and it assigns responsibility to the sunk consumer L7 on line 5-7, while no other method does, since the flow goes in an opposite sense to the consumer’s demand L7.

The absolute responsibility of consumer L4 in bus 4 over line 3-4 is clear, since this line is fully dedicated to this consumption. That situation is recognized by the different methods, except for the average participations method, see Table V and Figure 4.

### TABLE III

<table>
<thead>
<tr>
<th>METHOD</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
<th>L6</th>
<th>L7</th>
<th>L8</th>
<th>%TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHAPELEY VALUE</td>
<td>11.21%</td>
<td>18.13%</td>
<td>65.17%</td>
<td>2.49%</td>
<td>2.93%</td>
<td>0.11%</td>
<td>100%</td>
</tr>
<tr>
<td>GLDF</td>
<td>15.04%</td>
<td>17.96%</td>
<td>61.98%</td>
<td>4.16%</td>
<td>1.32%</td>
<td>0.21%</td>
<td>100%</td>
</tr>
<tr>
<td>AVERAGE PARTICIPATION</td>
<td>12.80%</td>
<td>28.36%</td>
<td>45.21%</td>
<td>2.93%</td>
<td>8.72%</td>
<td>1.98%</td>
<td>100%</td>
</tr>
<tr>
<td>KERNEL</td>
<td>11.08%</td>
<td>15.75%</td>
<td>64.18%</td>
<td>3.77%</td>
<td>3.26%</td>
<td>1.97%</td>
<td>100%</td>
</tr>
</tbody>
</table>

### TABLE IV

<table>
<thead>
<tr>
<th>METHOD</th>
<th>Line 1-2</th>
<th>Line 2-3</th>
<th>Line 3-4</th>
<th>Line 3-5</th>
<th>Line 3-6</th>
<th>Line 5-7</th>
<th>Line 6-7</th>
<th>Line 7-8</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHAPELEY VALUE</td>
<td>22.44%</td>
<td>14.91%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>GLDF</td>
<td>10.08%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>10.36%</td>
<td>20.00%</td>
</tr>
<tr>
<td>AVERAGE PARTICIPATION</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>47.11%</td>
<td>27.48%</td>
<td>27.48%</td>
<td>20.00%</td>
</tr>
<tr>
<td>KERNEL</td>
<td>20.23%</td>
<td>15.18%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>19.88%</td>
<td>20.00%</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

It is interesting to note the similarity of the results among the different compared methods, especially when recognizing the biggest and smaller responsibilities of the different loads in the costs of the transmission system.

The results obtained, by means of games theory formulation, allow to give solutions in lines where the traditional methodologies, like GLDF and average participations, do not give solutions or their values are very different.

The technical and economic modeling suggested for investment, electric network operation and maintenance cost allocation of transmission systems consider that the agent’s cooperation and coordination has been established by coalition formation. If the obtained solutions complete the conditions of rationality imposed in cooperative games, in other words, the solutions belong to the core of the game, the solutions are considered to be efficient, fair and equitable to participant agents. The transmission cost allocation method presented, is based on the cooperative game theory and transmission network capacity use by consumer agents.
GAME THEORY IN COST ALLOCATION

A cost allocation cooperative game is given by a couple (\(NA, C\)), where \(NA\) is the set of the \(N\) agents and \(C\) is the cost function. The agents can group in many different ways according to their interests and convenience. The way in which \(N\) players group in \(m\) mutually exclusive and excluding coalitions \(S\), is the coalition configuration.

\[
\delta = \{S_1, S_2, \ldots, S_m\}
\]

\(\delta\) is a partition of \(NA\) that fulfills three conditions.

\[
S_j \neq \Phi; j = 1,2, \ldots, m
\]

\[
S_j \cap S_i = \Phi; \forall i \neq j
\]

\[
\bigcup_{s=a}^{S_j} = NA
\]

where \(\Phi\) is the empty set.

Each agent belongs to one and only one of the \(m\) coalitions and the members of a certain coalition are related to each other but not with other agents that belong to other coalitions.

The costs assigned to each agent as a result of the game correspond to the vector of assignments or payments \(x = \{x_1, x_2, \ldots, x_N\}\), where \(x_i\) is the agent \(i\) payoff. The result of a game is the payoff configuration PC:

\[
PC = \{x; \delta\} = (x_1, x_2, \ldots, x_N; S_1, S_2, \ldots, S_m)
\]

The solution of the game should fulfill the conditions of rationality (individual, collective and global) and stability.

\[
x(i) \leq C(i); \forall i \in NA
\]

\[
x(S) \leq C(S); \forall S \notin \delta
\]

\[
x(NA) = C(NA)
\]

with:

\[
x(S) = \sum_{i \in S} x_i
\]

Any agent or coalition of agents should not have a bigger cost that their alternative cost or stand-alone cost. The resulting assignment of costs of the game to all the players must be identical to the total costs to be covered. This last condition is known as break-even condition or Pareto-optimum.

The payoffs that are Pareto-optimun and individually rational are called imputations. Besides, if it is collectively rational, the core of the game is obtained.

The core of the game \((NA, C)\) is the set of all the solutions PC \((x; \delta)\) such:

\[
x(T) \leq C(T); \forall T \subset NA, \text{ with: } x(T) = \sum_{i \in T} x_i
\]

The core is, therefore, a subset of the group of imputations. This core concept is the simplest of all the solution concepts of cooperative games. It corresponds to a group of imputations that leaves no space for a better allocation to its players and does not allow subsidies among coalitions. Unfortunately, there are many games with too large cores (many solutions), no core at all or empty core. The empty core takes place when the collective rational is not achieved.

Starting from the core it is possible to establish different theories that outline solutions to this type of games. Solutions could be: extensions of the core, stable set and bargaining set.

Other solutions can be achieved with the excess theory, defined as the difference between the coalition stand-alone cost and the payoff for that coalition as a result of the game. Thus, the excess of coalition \(R\) with respect to the payoff vector \(x\) of the PC \((x; \delta)\) is defined as:

\[
e_{R}(x) = C(R) - x(R)
\]

and represents the total amount that potential members of the \(R\) coalition collectively win or lose if they withdraw from \(R\).

From the excess definition it is possible to incorporate the Nucleolus and Kernel solution mechanisms.

The nucleolus solution corresponds to imputations for which the maximal excess is minimized:

\[
\text{max} \{\text{min} e(R)\}; \forall R
\]

If \(S_{kl}\) is the maximum surplus between agent “\(k\)” over agent “\(l\)”, with respect to a coalition configuration, then:

\[
S_{kl} = \text{Min}_{R/k \in R, r \in R} e(R)
\]

Agent “\(k\)” is considered to be “stronger” than agent “\(l\)” if:

\[
S_{kl} < S_{lk}, \text{ with } x_j < c(t)
\]

In particular, “\(k\)” and “\(l\)” are in equilibrium if any of the following relationships is satisfied:

\[
S_{kl} = S_{lk}
\]

\[
S_{kl} < S_{lk}, \text{ with } x_j = c(t)
\]

\[
S_{kl} > S_{lk}, \text{ with } x_k = c(k)
\]

Using the concept of equilibrium, the Kernel can be defined as the set of all coalition configurations (and their associated payments) where the agents are in equilibrium.

The Kernel, as costs assignment, shows same advantages: provides a unique solution for the process of coalition formation developed by the game, belongs to the core (if it exists) of the game fulfilling the three coalition rationalities, the Kernel assignment corresponds to strength equilibrium between the agents belonging to a coalition. Therefore, the Kernel is a fair assignment from the perspective of the negotiation process.

Other traditional solution is the Shapley Value, given by:

\[
\phi_i = \frac{\sum_{S \subset NA} (N - s)! (s - 1)! [C(S) - C(S - \{i\})]}{N!} \text{ for } i \in NA
\]

where:

\(N\): total number of players.
VIII. APPENDIX B

CHARACTERISTIC FUNCTION

<table>
<thead>
<tr>
<th>COALITION</th>
<th>VALUE</th>
<th>COALITION</th>
<th>VALUE</th>
<th>COALITION</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>[18]</td>
<td>186.922</td>
<td>[4,6,8]</td>
<td>308.511</td>
<td>[3,4,6,7,8]</td>
<td>455.87</td>
</tr>
<tr>
<td>[23]</td>
<td>272.087</td>
<td>[6,7,8]</td>
<td>186.922</td>
<td>[3,4,7,8]</td>
<td>455.87</td>
</tr>
<tr>
<td>[24]</td>
<td>250.365</td>
<td>[6,8]</td>
<td>382.845</td>
<td>[3,4,6,7,8]</td>
<td>455.87</td>
</tr>
<tr>
<td>[26]</td>
<td>0</td>
<td>[8]</td>
<td>0</td>
<td>[2,4,8]</td>
<td>509.269</td>
</tr>
<tr>
<td>[27]</td>
<td>404.568</td>
<td>[4,7,8]</td>
<td>186.922</td>
<td>[3,4,6,7,8]</td>
<td>455.87</td>
</tr>
<tr>
<td>[28]</td>
<td>250.365</td>
<td>[6,7,8]</td>
<td>382.845</td>
<td>[3,4,6,7,8]</td>
<td>455.87</td>
</tr>
<tr>
<td>[29]</td>
<td>401.428</td>
<td>[5,6,8]</td>
<td>401.428</td>
<td>[3,4,6,7,8]</td>
<td>253.251</td>
</tr>
<tr>
<td>[31]</td>
<td>272.087</td>
<td>[6,7,8]</td>
<td>186.922</td>
<td>[3,4,7,8]</td>
<td>455.87</td>
</tr>
<tr>
<td>[32]</td>
<td>250.365</td>
<td>[6,8]</td>
<td>382.845</td>
<td>[3,4,6,7,8]</td>
<td>455.87</td>
</tr>
<tr>
<td>[34]</td>
<td>0</td>
<td>[8]</td>
<td>0</td>
<td>[2,4,8]</td>
<td>509.269</td>
</tr>
</tbody>
</table>

IX. ACKNOWLEDGMENT

The authors gratefully acknowledge the support from Universidad de Santiago de Chile and Universidad Católica de Chile.

X. REFERENCES


XI. BIOGRAPHIES

Juan Zolezzi, (M’98), was born in Valdivia, Chile, and graduated as an Electrical Engineer from Technical State University of Chile, later obtaining his M.Sc. from University of Chile and Ph.D. from Catholic University of Chile. He is a professor at Santiago University of Chile. His research activities focus on power economics, economic transmission issues, planning and regulation of electric markets. He has been a consultant with the World Bank and with utilities and regulators in Europe.

Hug Rudnick, (F’00), was born in Santiago, Chile, and graduated as an Electrical Engineer from University of Chile, later obtaining his M.Sc. and Ph.D. degrees from the Victoria University of Manchester, UK. He is a professor at Catholic University of Chile. His research activities focus on the economic operation, planning and regulation of electric power systems. He has been a consultant with the World Bank and with utilities and regulators in Argentina, Bolivia, Brazil, Canada, Central America, Chile, Colombia, Mexico, Peru and Venezuela, as well as in Europe.