Finite set model predictive control to a shunt multilevel active filter

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Abstract
Purpose – The purpose of this paper is to implement a finite set model predictive control algorithm to a shunt (or parallel), multilevel (cascaded H-bridge) active power filter (APF). Specifically, the purpose is to get a controller that could compensate the mains current and, at the same time, to control the voltages of its capacitors. This strategy avoids the use of multiple PWM carriers or another type of special modulator, and requires a relatively low processing power.
Design/methodology/approach – This paper is focused in the application of the predictive controller to a single-phase parallel APF composed for two H-bridges connected in series. The same methodology can be applied to a three-phase APF. In the DC buses of each H-bridge, a floating capacitor was connected, whose voltage is regulated by the predictive controller. The controller is composed by, first, a model for the charge/discharge dynamics for each floating capacitor and a model for the output current of the APF; second, a cost function; and third, an optimization algorithm that is able to control all these variables at the same time, choosing in each sample period the best combination of firing pulses.
Findings – The controller can track the voltage references, compensate the current harmonics and compensate reactive power with an algorithm that evaluates only the three nearest voltage levels to the last voltage level applied in the inverter. This strategy decreases the number of calculations required by the predictive algorithm. This controller can be applied to the general case of a single-phase multilevel APF of N-levels and extend it to the three-phase case without major problems.
Research limitations/implications – The implemented controller, when the authors consider a constant sample time, gives a mains current with a Total Harmonic Distortion (THD-I) slightly greater in comparison with the base algorithm (that evaluates all the voltage levels). However, when the authors consider the processing times under the same processor, the implemented algorithm requires less time to get the optimal values, can get lower sampling times and then a best performance in terms of THD-I. To implement the controller in a three-phase APF, a faster Digital Signal Processor would be required.
Originality/value – The implemented solution uses a model for the charge/discharge of the capacitors and for the filter current that enable to operate the cascaded multilevel inverter with asymmetrical voltages while compensates the mains currents, with a predictive algorithm that requires a relatively low amount of calculations.
Keywords Electric converters, Electric power systems, Predictive techniques
Paper type Research paper

1. Introduction
The increasing use of electric drives and power supplies in the industrial and residential installations has produced power quality degradation in the electrical...
networks. The harmonic current injection and voltage distortion in the mains could produce problems as overheating in the components of the power system or their premature failure, among other undesirable effects. These problems have motivated the development of new concepts and technologies during the last years, to reduce and mitigate their causes (Akagi et al., 2007).

With the objective of accomplish power quality standards, there are many kind of solutions, some of them based in the use of passive filters and others more sophisticated using Flexible AC Technologies (FACTs), as active power filters (APF).

The APFs can adapt to changing operating conditions of the system and they can reach multiple objectives, such as harmonic elimination and reactive power compensation. The most common kind of APF is the shunt APF, based in voltage source inverters (two-level inverter).

Since the begin of the 1980 decade, many techniques for harmonic current extraction using APF have been proposed, some of them are time based (capacitor voltage control, currents correlation, instantaneous power or pq-theory, synchronous reference frame or dq-theory) and others are frequency based (Resonant controllers, Fourier series, wavelets) (Massoud et al., 2004; Szromba, 2004; Bojoi et al., 2008).

On the other side, the multilevel converters have become very popular in high-power and high-voltage applications because they can produce better quality waveforms (lower THD) using standard semiconductors (Wu, 2006; Franquelo et al., 2008). The most popular multilevel inverter configurations are: Neutral Point Clamped (NPC), Flying Capacitor (FC) and Cascaded H-Bridge (CHB) (Rodrı ´ guez et al., 2002). Among these multilevel converter topologies, the CHB is very attractive for its modularity and the ability of reach high-power levels using standard semiconductors.

The main drawback of this converter is that requires one isolated voltage source for each H-bridge Cell. Other advantage of this converter topology is the possibility to get more voltage levels using asymmetrical voltage sources, at the cost of losing redundancies and his modularity (Dixon and Moran, 2006).

Various control strategies for CHB multilevel APFs have been proposed, many of them based on the generation of multiple PWM carriers for controlling each H-bridge and keep the voltages in balance (symmetric voltage operation) (Jianlin et al., 2003; Zhou et al., 2004; Rani and Porkumaran, 2010; Karuppanan and Mahapatra, 2010; Madhukar and Agarwal, 2010). Other methods are based on multiple control loops to get asymmetric voltages in the H-bridges (Lopez et al., 2003; Pérez-Ramı´rez et al., 2010).

Another kind of control algorithm for these systems is the predictive current control, which consists in using a model of the system to get the optimal input signal. In Massoud and Williams (2004) the model is used to get the optimal voltage vector (in the αβ frame) and then it is introduced in a Space Vector Modulator. In Odavic et al. (2011) the authors propose a predictive control algorithm to get the optimal voltage vector too, but they use a modulator based in multiple PWM carriers. Both works implement a converter operating with symmetrical voltages.

Another approach in predictive algorithms is the so called Finite Set Model Predictive Control (FS-MPC). It consists on solving an optimization problem in real time to get the optimal control action to apply in the next sampling period, having in consideration the discrete nature of the static converters. This idea has been proposed in Pontt et al. (2007) for a voltage source inverter. Among the advantages of this approach could be mentioned: first, dynamic responses comparable with the conventional controllers; second, eliminates the modulator because the algorithm gives the optimal combination of firing pulses for the transistors; and third, allow to
include multiple control objectives and restrictions in an intuitive and easy form (Cortés et al., 2008). The main drawback of this control strategy is that requires a fast sampling and processing system to solve the problem in real time. There are some works that had used this approach in many power converter applications, like Active Front-End rectifiers, Voltage Source Inverters, Matrix Converters and motor Drives (Rodríguez et al., 2013). In Vyncke et al. (2012) the authors analyze some design alternatives in the FS-MPC algorithm for a FC Converter, such as simplification of the model, selection of the weight factors and increasing the prediction horizon. There are some works where a major reduction of the control set is proposed at the expense of achieving suboptimal solutions like in Duran et al. (2011) where a Restrained Search Predictive Control (RSPC) is applied for a Dual Three Phase Drive and in Cortes et al. (2009) a FS-MPC strategy for a multilevel CHB inverter where only the adjacent vectors to the last applied (in the αβ frame) are considered in the optimization stage. For this all the adjacent vectors must be off-line computed in a look-up table.

An application of FS-MPC for a CHB multilevel APF with four symmetric cells (four H-bridges in series) is shown in Paredes et al. (2010), however, the authors only control the APF current while the DC buses of the H-bridges are connected to isolated voltage sources.

This document presents an application of model predictive control to a parallel/shunt CHB APF (consisting in two asymmetric H-bridges in series) that compensates the mains current while controls the voltage in the two floating capacitors, which are in 3:1 ratio to get a maximum of nine levels. This is possible because each voltage control is relatively independent of the other, giving the possibility of operate the inverter asymmetrically to get more voltage levels in the output.

The proposed algorithm uses a model for the APF output current and one model for the each capacitor (charge/discharge dynamics) in function of switching signals and the APF output current, and then it solves the optimization problem using only the three nearest voltage levels of the last voltage applied. With this strategy, the amount of calculations is notably reduced. The control system does not need a modulator or additional control loops to control the voltage asymmetries.

2. System description

Figure 1 shows the system, which consists in a single-phase shunt/parallel APF. The APF is composed of a CHB multilevel inverter and with floating capacitors connected in its DC buses. The inverter is connected to the Point of Common Connection with an inductor (whose inductance and Resistance is $L_f$ and $R_f$, respectively). The predictive controller allows the APF to follow its current reference (given by the current reference generator), and capacitors voltage references, whose values are fixed and are in 3:1 ratio to get a maximum of nine voltage levels in the output.

3. Multilevel inverter

The multilevel inverter is composed of two H-bridges connected in series as can be seen in the Figure 4. The H-bridge that operates with the higher voltage is called Main, while the H-bridge with the lower voltage is called Auxiliary. A floating capacitor is connected in the DC bus of each H-bridge ($C_m$ and $C_a$) and their voltages are controlled to keep a ratio of $v_{cm}/v_{ca} = 3:1$, to get a maximum of nine voltage levels in the output.

The Table II shows the non-redundant combinations of firing pulses that generate the nine voltage levels. These combinations have been selected arbitrarily.
4. Generation of APF current reference
The current reference for the APF is obtained using a well-known approach that consists in generating the mains current reference using a PI controller, which controls the error in the sum of voltages in the floating capacitors. This scheme can be seen in the Figure 7.

5. FS-MPC
The Figure 5 shows a flow diagram for the controller, which is composed of an algorithm that sample the signals \([i \_s(k); i \_f(k); v \_s(k); v \_cm(k); v \_ca(k)]\), then using these values and a model for the system, get predictions for the controlled variables \([i \_f(k+1); v \_cm(k+1); v \_ca(k+1)]\) for each firing pulses combination a given set. These predicted variables are compared with their references through a cost function that increases as these values differ. Next, the algorithm chooses the firing pulses combination that minimizes the cost function and finally the outputs for the controller are refreshed with this firing pulses combination and then the procedure is repeated. In the next sections a more detailed explanation for each component of the algorithm is given.
5.1 Multilevel inverter model

To get the model for the capacitor dynamics, we can first consider the simple H-bridge with a capacitor connected in the DC bus shown in the Figure 2. In this case, for describe the behavior of the system through equations; we had reserved the letter S to represent the transistor’s firing signals, which is defined in the following form:

\[
S_j = \begin{cases} 
1 & \text{Transistor } T_j \text{ is in ON state} \\
0 & \text{Transistor } T_j \text{ is in OFF state}
\end{cases}
\]

We must note that this firing signal only tells us if the transistor is ON or OFF, but it not necessarily mean that the current is going through the transistor.

In each leg of the H-Bridge we have two transistors, labeled \(T_x\) and \(T'_x\) \((x = 1, 2)\) whose firing states \(S_x\) and \(S'_x\) are complementary to avoid a short-circuit. For this fact we can only consider the upper firing signal without loss of generality in our analysis.

Considering these facts we have eight possible states for the Charge/Discharge of the capacitor when the H-Bridge fed a load, showed in the Figure 3, where the path of the current has been highlighted for each state of the converter. The figure also shows the fundamental output voltage the load current (assuming that it is sinusoidal) to illustrate the current/output voltage possible states.

We can note that the capacitor dynamics depends as much on the firing signals \(S_1\) and \(S_2\) as it does on the direction of the load current \(i_\text{o}\), because in some cases even though a transistor is ON, it does not conduct the load current but for their antiparallel diode. These results are been summarized in the Table I.

By mean of this analysis we can note that the output voltage and the capacitor current can be obtained as follows:

\[
v_{\text{OUT}}(t) = v_c(t) \cdot [S_1 \cdot \overline{S_2} - \overline{S_1} \cdot S_2]
\]

\[
i_C(t) = C \frac{dv_c(t)}{dt} = -i_\text{o}(t) \cdot [S_1 \cdot \overline{S_2} - \overline{S_1} \cdot S_2]
\]

where we have considered \(\overline{S}_x = (1 - S_x)\) for write the equations in a simpler form and to emphasize the binary nature of the firing signals. The last equation could be used for modeling the capacitor dynamics in function of load current and the firing signals.

Now, for the multilevel inverter shown in Figure 4 the output voltage is given by the Equation (3), where \(S_{1m}\) with \(S_{2m}\) and \(S_{1a}\) with \(S_{2a}\) are the firing signals for the transistors of the Main H-bridge and the Auxiliary H-bridge, respectively:

\[
v_{\text{OUT}} = v_{cm} \cdot (S_{1m} \cdot \overline{S_{2m}} - \overline{S_{1m}} \cdot S_{2m}) + v_{ca} \cdot (S_{1a} \cdot \overline{S_{2a}} - \overline{S_{1a}} \cdot S_{2a})
\]
On the other side, the relation that governs the charge and discharge of the capacitors connected to each H-bridge are given by:

\[ C_m \frac{dv_{c_m}}{dt} = -i_f(t) \cdot (S_{1_m} \cdot S_{2_m} - S_{1_m} \cdot S_{2_m}) \]

\[ C_a \frac{dv_{c_a}}{dt} = -i_f(t) \cdot (S_{1_a} \cdot S_{2_a} - S_{1_a} \cdot S_{2_a}) \] (4)

Figure 3. Charge/discharge dynamics of the capacitor when the H-Bridge feeds a load.
If we applied the forward Euler approximation for the derivative, using a sample period $T_s$, we get the following relations:

$$v_{C_m}(k+1) = v_{C_m}(k) - \frac{T_s}{C_m} i_f(t) \cdot \left( S_{1_m} \cdot S_{2_m} - S_{1_m} \cdot S_{2_m} \right)$$

$$v_{C_a}(k+1) = v_{C_a}(k) - \frac{T_s}{C_a} i_f \cdot \left( S_{1_a} \cdot S_{2_a} - S_{1_a} \cdot S_{2_a} \right)$$

(5)

Those relations correspond to the equations that describe the capacitors dynamics for discrete-time.

**Note:** The fundamental output of the output voltage, the output current and the instantaneous output voltage are shown.

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**Figure 3.**
5.2 APF current model

Starting with the equation that describes the dynamic for the APF inductor current:

\[ v_s(t) - v_{out}(t) = L_f \frac{di_f(t)}{dt} + R_f \cdot i_f(t) \]  

(6)

And applying the Euler approximation for the derivative using a sampling time of \( T_s \), we get:

\[ i_f(k + 1) = \left(1 - \frac{R_f T_s}{L_f}\right)i_f(k) + \frac{T_s}{L_f}(v_s(k) - v_{out}(k)) \]  

(7)

That corresponds to the dynamic equation for discrete-time for the APF current.

5.3 Control sets considered

As the APF current and the voltage of the capacitors depend on the combination of the firing signals \([S1_m; S2_m; S1_a; S2_a]\), the total number of combinations is 16. However, as

<table>
<thead>
<tr>
<th>Firing (control) signals</th>
<th>Output current sign (i_o)</th>
<th>Active (conducting) semiconductor</th>
<th>Output voltage (V_{out})</th>
<th>Change on (vc) (dv_c/dt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>S2</td>
<td>(&lt;0)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>(&gt;0)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>(&gt;0)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>(&gt;0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>(&lt;0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>(&lt;0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>(&lt;0)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table I.

Output voltage and active semiconductor in function of firing pulses and load current sign

Figure 4.

Two cells, cascaded H-bridge multilevel inverter
the inverter operates with voltages in 3:1 ratio, the total number of non-redundant combinations for the firing signals is nine.

For the selection of the firing signals combinations (S) to consider in the optimization algorithm, two alternatives were used: first, evaluate all the nine non-redundant firing signals; and second, only consider the firing pulse combinations that give the three nearest voltages levels from the last voltage applied.

To give a better understanding of the last alternative, assume that we have a N-Level converter whose voltage difference between two adjacent levels is $V_{dc}$, then, we can define the set of the feasible voltages in its output as $[V_0; V_1; V_2; \ldots; V_{N-2}; V_{N-1}]$ where the subscript represents the multiple of $V_{dc}$ applied in that state.

The algorithm works as follows: if the last voltage level applied in the inverter was $V_j$, the optimization algorithm only will evaluate the cost function for the three nearest voltage levels to $V_j$, i.e. $[V_{j-1}; V_j; V_{j+1}]$. In the case that the last voltage applied was $V_0$, the following three voltages are used $[V_0; V_1; V_2]$ and when the last voltage applied was $V_{N-1}$, the following three voltages are used $[V_{N-3}; V_{N-2}; V_{N-1}]$.

In the Figure 8, a comparison between the two alternatives for the selection of the set of firing signals to be considered in the evaluation of the cost function is shown by a flow diagram.

5.4 Cost function
To reach these control objectives, the following cost function is defined, which is evaluated for every combination of firing pulses in each control set considered:

$$J_N(S) = |i^*_f - i_f(k+1|k)| + \lambda|V^*_{cm} - V_{cm}(k+1|k)| + \gamma|V^*_{ca} - V_{ca}(k+1|k)|$$  \hspace{1cm} (8)

where $V^*_{cm}$, $V^*_{ca}$ and $i^*_f$ are the reference values for the voltages in the capacitors and the APF current, respectively. $V_{cm}(k+1|k)$, $V_{ca}(k+1|k)$ correspond to the predicted values for the voltages in the capacitor of the Main and the Auxiliary H-bridges and $i_f(k+1|k)$ is the predicted value for the APF current for the next sampling period, and their values are given by the Equations (5) and (7), respectively.

$\lambda$ and $\gamma$ correspond to the multipliers for the cost function and they enable the inclusion of multiple control objectives in the predictive controller algorithm. Their values were found following the procedure and guidelines in Cortes et al. (2009) with the help of simulations, where were evaluated the Mean Squared of the reference tracking Errors for $i_s$, $v_{cm}$ and $v_{ca}$ besides the Total Harmonic Distortion (THD) in the mains current and APF output voltage.

5.5 Extension to the three-phase case
To extend the control algorithm to the three-phase case, the system can be considered as three single-phase APF working independently both in the current reference generation and in the predictive algorithm if we consider a three wire system and work with phase to phase voltages. In this case, only the second alternative for the selection of firing pulses (control set) was considered, because the first alternative is impractical due to the computational burden required. In the second alternative for the selection of the control set, the number of firing signals combinations considered is $3^3 = 27$ (three for each phase) (Figure 5).
In this case, considering a three-conductor system, the equations that describe the dynamics of the system (shown in the Figure 6) are:

\[
v_{abf}(t) - L_{fa} \frac{di_{af}(t)}{dt} - i_{af}(t)R_{fa} + L_{fb} \frac{di_{bf}(t)}{dt} + i_{bf}(t)R_{fb} - e_{ab} = 0
\]

\[
v_{bcf}(t) - L_{fb} \frac{di_{bf}(t)}{dt} - i_{bf}(t)R_{fb} + L_{fc} \frac{di_{cf}(t)}{dt} + i_{cf}(t)R_{fc} - e_{bc} = 0
\]

\[i_{af}(t) + i_{bf}(t) + i_{cf}(t) = 0\]  \hspace{1cm} (9)

Now assuming that the resistance and inductance of the inductor is the same for all phases (i.e. \(R_{fa} = R_{fb} = R_{fc} = R\) and \(L_{fa} = L_{fb} = L_{fc} = L\)). Applying the forward Euler approximation, the above relations in discrete-time equations become:

\[
\begin{bmatrix}
i_{af} \\
i_{cf}
\end{bmatrix}_{k+1} = \left(1 - \frac{RT_S}{L}\right)\begin{bmatrix}
i_{af} \\
i_{cf}
\end{bmatrix}_k + \frac{T_S}{3L} \begin{bmatrix}
-2 & -1 \\
1 & 2
\end{bmatrix}\left(\begin{bmatrix}
v_{abf} \\
v_{bcf}
\end{bmatrix}_k - \begin{bmatrix}
e_{ab} \\
e_{bc}
\end{bmatrix}_k\right)
\]

\[
\begin{bmatrix}
i_{bf}
\end{bmatrix}_{k+1} = -\begin{bmatrix}
i_{af} \\
i_{cf}
\end{bmatrix}_{k+1} - \begin{bmatrix}
i_{cf}
\end{bmatrix}_{k+1}
\]

\hspace{1cm} (10)
And the cost function is given by:

$$J_N(S) = \sum_{n=a,b,c} |i_{fn}^* - i_{fn}(k + 1|k)| + \lambda |V_{c1n}^* - V_{c1n}(k + 1|k)| + \gamma |V_{c2n}^* - V_{c2n}(k + 1|k)|$$

(11)

where the APF output currents and capacitor voltages are calculated in a similar way as the single-phase case. As we can see, the principal difference between this equation and the cost function for a single-phase system (Equation (6)) is the inclusion of the other two phases.

6. Simulations

MATLAB – Simulink with the SimPowerSystems Toolbox was used to simulate the application of the predictive controller to the described system and find the values for the cost-function multipliers (Table II).

The simulation parameters are showed in the Table III, while the values for $\lambda$ and $\gamma$ for each simulation case are listed in the Table VI.

To compare the multilevel APF performance, a three-level APF was simulated. This simulation can be obtained by bypassing one of the H-bridges of the multilevel inverter and changing the voltage reference for the capacitor.

The value of the multipliers was found running a set of simulations with different values for $\lambda$ and $\gamma$ to get the merit values for the APF as be mentioned previously. A brief list of values for the weighting factor and the merit values for the APF can be seen in the Table IV for the case with the extended control set and in Table V for the case of reduced control set. We can note that in both cases the values for the weighting factor for the main capacitor must be higher in one order of magnitude than the weighting factor for the auxiliary capacitor. This fact could be explained because the main capacitor has the main proportion of the voltage (and thus of
power). We also have to say that in the neighborhood of the selected values for $\lambda$ and $\gamma$ (i.e. $\pm 20$ percent), the performance of the control algorithm (in terms of stability and the harmonic distortion) is not affected considerably. In the other hand, as mentioned in Pontt et al. (2007), errors in the same proportion ($\pm 20$ percent) in the values used in

<table>
<thead>
<tr>
<th>S1m</th>
<th>S2m</th>
<th>S1a</th>
<th>S2a</th>
<th>VOUT</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$-4V_{DC}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$-3V_{DC}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$-2V_{DC}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$-V_{DC}$</td>
</tr>
</tbody>
</table>

Table II. Non-redundant combinations of firing pulses used in the two-cells CHB

Table III. Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductance $L_f$</td>
<td>$6 \text{ m (H)}$</td>
</tr>
<tr>
<td>Resistance $R_f$</td>
<td>$0.8 \text{ (\Omega)}$</td>
</tr>
<tr>
<td>Capacitance $C_m$ and $C_a$</td>
<td>$2,200 \text{ \mu (F)}$</td>
</tr>
<tr>
<td>Mains peak voltage $v_x$</td>
<td>$100 \text{ (V)}$</td>
</tr>
<tr>
<td>Load resistance $R_D$</td>
<td>$15 \text{ (\Omega)}$</td>
</tr>
<tr>
<td>Load inductance $L_D$</td>
<td>$30 \text{ m (H)}$</td>
</tr>
</tbody>
</table>

Table IV. Influence of the weight factors on the merit variables for the nine level APF

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\gamma$</th>
<th>$MSE I_s$ (A)</th>
<th>$MSE v_{cm}$ (V)</th>
<th>$MSE v_{ca}$ (V)</th>
<th>THD-$I_s$ (%)</th>
<th>THD-$V_0$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>0.441</td>
<td>27.066</td>
<td>27.328</td>
<td>6.40</td>
<td>37.94</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>0.433</td>
<td>20.887</td>
<td>21.164</td>
<td>6.38</td>
<td>32.46</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
<td>0.424</td>
<td>0.385</td>
<td>0.877</td>
<td>6.27</td>
<td>20.61</td>
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<tr>
<td>100</td>
<td>10</td>
<td>0.432</td>
<td>0.161</td>
<td>0.804</td>
<td>6.38</td>
<td>22.89</td>
</tr>
<tr>
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<td>20</td>
<td>0.473</td>
<td>0.066</td>
<td>0.768</td>
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<td>27.80</td>
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<tr>
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<td>30</td>
<td>0.479</td>
<td>0.041</td>
<td>0.757</td>
<td>7.64</td>
<td>37.99</td>
</tr>
<tr>
<td>400</td>
<td>40</td>
<td>0.495</td>
<td>0.079</td>
<td>0.793</td>
<td>7.99</td>
<td>43.48</td>
</tr>
</tbody>
</table>

Table V. Influence of the weight factors on the merit variables for the nine level APF with the reduced control set

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\gamma$</th>
<th>$MSE I_s$ (A)</th>
<th>$MSE v_{cm}$ (V)</th>
<th>$MSE v_{ca}$ (V)</th>
<th>THD-$I_s$ (%)</th>
<th>THD-$V_0$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>0.875</td>
<td>18.786</td>
<td>19.809</td>
<td>66.57</td>
<td>42.95</td>
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<tr>
<td>20</td>
<td>4</td>
<td>0.909</td>
<td>24.628</td>
<td>31.035</td>
<td>48.70</td>
<td>38.95</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>0.884</td>
<td>4.252</td>
<td>7.780</td>
<td>19.80</td>
<td>44.30</td>
</tr>
<tr>
<td>100</td>
<td>15</td>
<td>0.908</td>
<td>2.349</td>
<td>3.131</td>
<td>9.98</td>
<td>38.67</td>
</tr>
<tr>
<td>150</td>
<td>20</td>
<td>0.984</td>
<td>9.149</td>
<td>5.706</td>
<td>13.93</td>
<td>41.50</td>
</tr>
<tr>
<td>300</td>
<td>25</td>
<td>1.487</td>
<td>13.068</td>
<td>7.996</td>
<td>22.15</td>
<td>51.14</td>
</tr>
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</table>
the algorithm for the resistance and – mainly – in the inductance of the coupling inductor \((R_F, L_F)\) do not have significant effects in the current tracking dynamics (Figures 7 and 8; Table VI).

The results for the simulations of the single-phase three-level APF are shown in the Figures 9 and 10, where the Figure 9 shows the voltage in the capacitor and the output voltage of the inverted, and the Figure 10 shows the current of the non-linear load, the APF current and the mains current.

Figures 11 and 12 show the voltages and currents for the single-phase nine-level inverter when all the nine feasible voltages levels are considered in the optimization algorithm, while the Figures 13 and 14 show the voltages and currents for the case when only the three nearest voltage levels are considered in the optimization process.

The results of the simulations, for a sampling period of \(T_s = 40 \mu s\) and \(T_s = 100 \mu s\), in terms of Total Harmonic Distortion for the APF output Voltage

<table>
<thead>
<tr>
<th>Case</th>
<th>(\lambda)</th>
<th>(\gamma)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Level</td>
<td>1</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>9 Level</td>
<td>300</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>9 Level, Nearest 3</td>
<td>100</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

Notes: (a) All the voltage levels; (b) the nearest three-levels

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Figure 7.
Current reference generation scheme for the single-phase APF

Figure 8.
Comparison of the schemes of set of firing signals selection

Table VI.
Multipliers used for each cost function
(THD-Vo), and Total Harmonic Distortion in the mains Current (THD-Is) are listed in the Table VII. We have to say that the THD-Vo is relatively more important because the THD-Is also depends on the coupling inductance $L_f$.

Simulation results show that in the case of three-level APF and the nine-level APF when the extended control set is considered, when we increase the sample period, the THD-Is as the THD-Vo increases their value, which could be explained due to that an increase on the sampling period implies a lower capacity to follow the current reference thus the algorithm selects distant voltage levels (i.e. non-adjacent) producing higher distortion.

On the other hand, when we increase the sample period for the nine-level APF when the algorithm only uses the three nearest voltage levels, only the THD-Is shows an increase (<4 percent) while the THD-Vo decreases in near 8 percent. This fact could be
explained due to that although the controller cannot follow the current reference, the restriction of select only the nearest voltage levels limits the distortion in the output voltage.

Figures 15 and 16 show the results for the three-phase case with the algorithm that uses only the three nearest voltage levels (per-phase). It can be noted that the results are comparable with the single-phase case. We must say that follow the same
Figure 13. Voltages for the nine-level APF when only the three nearest voltage levels are considered for $T_s = 40 \mu s$.

Notes: (Upper) voltage in main capacitor; (center) voltage in the auxiliary capacitor; (bottom) comparison between the mains voltage and the output voltage of the active filter.

Figure 14. Currents for the nine-level APF when only the three nearest voltage levels are considered for $T_s = 40 \mu s$.

Notes: (Upper) load-distorted current; (center) APF output current; (bottom) compensated current.

Table VII. Total harmonic distortion for the line current (THD-Is) and for the converter voltage (THD-Vo) obtained in simulations.

<table>
<thead>
<tr>
<th>Case</th>
<th>THD-Is (%)</th>
<th>THD-Vo (%)</th>
<th>THD-Is (%)</th>
<th>THD-Vo (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Level</td>
<td>7.68</td>
<td>60.32</td>
<td>10.28</td>
<td>69.08</td>
</tr>
<tr>
<td>9 Level</td>
<td>7.64</td>
<td>37.99</td>
<td>8.84</td>
<td>39.38</td>
</tr>
<tr>
<td>9 Level, Nearest 3</td>
<td>9.98</td>
<td>38.67</td>
<td>13.69</td>
<td>30.31</td>
</tr>
</tbody>
</table>
APF current reference generation and controller, i.e. considering three single phase and independent APF, could result in a less stable system than one that consider the interactions and inter-phase dependencies (as a pq theory-based reference generator plus a three-phase FS-MPC algorithm).
7. Experimental results

With the purpose of validating the simulation results obtained; the predictive controllers were implemented in a reduced power prototype. The algorithms were implemented with the Texas Instruments F28335 EzDSP using Real-Time Workshop and Embedded Coder MATLAB’s Toolboxes. We must note that the DSP used have a limited computational capabilities (Floating point @150 MHz), and some program blocks were optimized after the MATLAB-assisted code generation.

Figure 17 shows the results obtained with the three-level APF case. The minimum sample period obtained for the controller algorithm was almost 30 $\mu$s, where signal sampling takes 5 $\mu$s, and 25 $\mu$s (3,750 instruction cycles) for the rest of the algorithm. As point of comparison, a PID algorithm (plus current reference generation) to control an APF with this DSP can take approximately 15-20 $\mu$s (2,250-3,000 instructions, considering the DSP running at 150 MHz).

In the figure we can see that the non-linear load current is compensated adequately but the APF current is “a bit noisy.”

Figure 18 shows the results for the case of the nine-level APF when all the voltage levels are used in the optimization algorithm. The DSP takes 126 $\mu$s that is more than
four times higher than in the three-level case. In this case the signal sampling is done in 6 \( \mu \)s, while the control algorithm takes 120 \( \mu \)s (18,000 instruction cycles).

As we can see, although the APF current is less noisy, higher commutation spikes appear in the compensated current due to the higher sample period.

Finally, the Figure 19 shows the results for the nine-level APF when only the three nearest voltage levels are considered in the optimization algorithm. The minimum sampling time reached was 56 \( \mu \)s that is only 87 percent higher than in the three-level case and less than a half of the time when the algorithm consider the extended control set. Here the sampling process, as in the last case, is done in 6 \( \mu \)s, but the control algorithm takes only 50 \( \mu \)s (7,500 instruction cycles).

Due to this savings in processing time, we could see that the commutation spikes in the compensated current are reduced.

8. Conclusion
This work has showed that: first, is possible to implement a Finite-Set Model Predictive Controller to a CHB Multilevel APF with asymmetric voltages achieving to compensate the mains current and control the capacitor voltages keeping a 3:1 ratio; second, a model for the charge/discharge dynamics for a capacitor connected in the DC bus of an H-bridge; and third, is possible to reduce considerably the computational burden required for the predictive algorithm in an easy way reducing the control set to

Notes: (Bottom in magenta) APF output voltage-scale 100V/Div (center in yellow) APF output current-scale 2A/Div--; (center in cyan) load current-scale 5A/Div--; (upper in red) compensated current-scale 5A/Div--

Figure 19. Currents and output voltage for the nine-level APF when the algorithm uses only the three nearest voltage levels, \( T_s = 56 \mu \)s
the three nearest voltage levels to the last one applied. Due its nature, this solution is applicable to converters with more levels (i.e. more H-Bridge Cells) without major changes.

The algorithms were simulated and implemented in a reduced power prototype showing satisfactory results.

Due to the limited processing capabilities of the DSP used in this work, only was possible to implement the algorithm for the single-phase case. However, nowadays there are available more powerful DSP and other technologies, like FPGA, for computational demanding applications.

References


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