Since shunt active power filters operate as controlled current sources injecting current harmonic components to the power distribution system, the point of connection must be carefully selected so the generated harmonic components flow to the nonlinear loads and do not propagate through the distribution system. In this article a numerical analysis based on analytical formulation that uses the power distribution system voltage and current transfer matrices is derived providing a user friendly and general tool to determine the most adequate point of connection for shunt active filters, improving the compensation effectiveness. Moreover, the proposed method allows the technical evaluation of the shunt active compensation in all the power distribution system. The validity of the developed method is verified by theory and computer simulation. Application to a real multibus power distribution system is presented.
Active Power Filters

Active power filters have become an interesting and effective solution for dynamic reactive power and current harmonic compensation in power distribution systems [1]. With the development of flexible ac transmission systems (FACTS) and flexible, reliable, and intelligent electrical energy delivery systems (FRIENDS) concepts in transmission and distribution [2], active filters will play an important role in the compensation, performance, and the power quality associated with such systems. Shunt active power filters have been used to compensate current harmonics and reactive power in power distribution systems, while series compensation has become the standard procedure for dynamic voltage compensation to eliminate sags and swells. Hybrid compensation has been considered as shunt active compensation since it operates absorbing current harmonics [1].

So far, active power filters have been analyzed in terms of principles of operation, control requirements, and compensation characteristics [3]–[6]. Compensation performance in real power distribution systems has been discussed in terms of the results obtained in voltage compensation, power factor correction, and current filtering for a specific load [7]–[8]. Few analyses can be found related with active power filter compensation performance in large power distribution systems [9]–[10].

The main objective of shunt active power filters is the elimination of current components that affect power distribution efficiency, such as harmonics and reactive components. If these current components are supplied by active power filters connected in a strategic bus, it is possible to confine the circulation of these unwanted current components in a specific region of the power distribution system; therefore improving the overall system efficiency and reliability. The selection of the active filter point of connection in multibus power distribution systems is not trivial and can affect current and voltage compensation performance significantly.

Akagi showed in [7] that compensation performance of shunt active power filters depends on the load characteristics and demonstrated that shunt active filters are more suitable for the compensation of current source type of loads. The compensation performance for voltage source type of loads is not fully satisfied with shunt compensation. This type of analysis [7] can be easily done in small power distribution systems or in equivalent circuit represented by the voltage source, the Thévenin equivalent impedance, and the nonlinear load. In real power distribution systems composed of a large number of buses and different types of loads, however, the compensation performance and effectiveness of a shunt active power filter strongly depends on the point of connection. It will be demonstrated that the selection of this point is not obvious and must be done carefully by considering the power system topology with the associated impedance values and the nonlinear load distribution. It is important to note that if the shunt active power filter is not located properly, it will contribute to increase the current and voltage distortion by injecting current harmonics that will circulate in all the distribution system. An inappropriate selection of the point of connection will be translated in a larger active filter rated power in order to obtain the same compensation effectiveness as compared with its connection in the proper bus.

In this article, we develop a procedure based on the power distribution current and voltage transfer matrices to exactly determine the more effective point of connection of a shunt active power filter. The proposed method is based on circuit equations and applied to multibus power distribution systems; the advantage is that it can be applied to any power distribution system configuration, since it uses standard mathematical algorithms normally employed in power system analysis. Further analysis of the results obtained with the proposed procedure allows to evaluate the compensation effectiveness in all distribution buses and the associated voltage sensitivity for active current compensation in different distribution nodes. Finally, simulated results prove the advantages of the proposed method and its effectiveness when it is applied to a power distribution system composed of a large number of buses.

Basic Equations

A power distribution system can be represented by a set of algebraic equations in the frequency domain. Each equation represents the relation between a number of dependent variables $X(\omega)$ with equal number of current sources $I(\omega)$, and also with voltage sources $V(\omega)$ through two power distribution system transfer matrices $H_{node}$ and $H_{nodeV}$, respectively.

The bus currents $I_{bus}$ are equal to the Norton equivalent current ($I_{Norton}$) plus the currents generated by the nonlinear loads ($I_{NL}$) and the currents supplied by controlled current sources, such as the one injected by shunt active power filters. Each of these currents is affected by their respective incident matrix. It is well known that

$$Y_{bus} \ast V_{bus} = I_{bus}. \quad (1)$$

This equation can be represented in terms of the different current components multiplied by the respective incident matrix, $A$, that is
The following branch matrices are defined:

\[ H_{\text{branch}} = Y_p * A_{\text{branch}} * H_{\text{model}} \]
\[ H_{\text{branchV}} = Y_p * A_{\text{branch}} * H_{\text{nodeV}} + Y_p. \]

Finally the branch current can be obtained from

\[ I_{\text{branch}} = H_{\text{branch}} * I_{\text{NL}} + H_{\text{branchV}} * V_p. \]

The current matrices \( H_{\text{model}} \) and \( H_{\text{branch}} \) relate the branch voltage and branch current that are induced when a current is injected in a different bus of the power distribution system. This characteristic is very useful for the analysis of active compensation effectiveness in a multibus power distribution system, since by solving (14), the most effective active power filter point of connection can be found, and also, for this specific point of connection, the effects in current and voltage distortion in all the other buses of the power distribution system can be obtained.

The method presented here is valid for passive and active filters analysis. The best point of connection for passive filters will be defined by avoiding resonances created with inductive elements connected in the power system. In the case of active power filtering, however, the compensation effectiveness is measured by evaluating the voltage distortion in different power system nodes and by calculating the required compensation current. In case the active power filter is not connected to the proper bus, the voltage harmonic distortion in the power systems might increase, since the current harmonics will not circulate to the nonlinear load, and will propagate to the rest of the power distribution system.

In our analysis, it is assumed that the control scheme that drives the active filter allows full compensation of the system current harmonic components. This assumption is valid since all the control schemes that have been proposed force the shunt active filter to operate as a current follower, therefore acting as a controlled current source with an acceptable frequency bandwidth.

**Deriving the Operating Characteristics of a Power Distribution System**

The solution of the branch transfer matrices shown in (12) and (13) allows evaluation of the operating characteristics of a power distribution system. In fact, the analysis of the system resonances can be done by computing \( H_{\text{model}} \) with all the current sources connected to the power distribution system and by plotting each matrix column element for different frequency values. This procedure finds the system resonant frequencies and the buses that are more affected. The system current transfer matrix \( H_{\text{model}} \) helps to determine the voltage sensitivity of a branch. This important characteristic can be used to find the bus that increases shunt active compensation effectiveness by reducing the associated \( \text{THD}_v \) in a power distribution system. This analysis can be done by deriving \( H_{\text{model}} \) for each bus in which the shunt active filter can be connected. Once \( H_{\text{model}} \) is obtained, the value of each row element for different frequency values must be calculated. The analysis of the matrix row elements as a function of
the frequency allows finding the buses in which the voltage is more affected by the active filter current injection. This result helps to determine the active power filter best point of connection.

The procedure described earlier is explained in more detail in the following example. A small power distribution system composed of three buses and six branches is shown in Figure 1.

The two current sources $I_1$ and $I_2$ represent the nonlinear loads. The primitive admittance matrix, $Y_p$, of the power distribution system shown in Figure 1 is equal to (15), shown at the bottom of the page, where the matrix elements are expressed in per unit with respect to 13.8 kV and 10 MVA base values and 50 Hz. The system branch incident matrix is equal to

$$A_{\text{branch}} = \begin{bmatrix} -1 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}. \quad (16)$$

The dimension of the branch incident matrix is $r \times n$, where $r$ is equal to the number of branches (6) and $n$ is equal to the number of buses of the power distribution system (3). The $Y_{\text{bus}}$ matrix is equal to

$$Y_{\text{bus}} = A_{\text{branch}}^T \cdot Y_p \cdot A_{\text{branch}}. \quad (17)$$

In this case, see (18) at the bottom of the page. The incident matrix of the existing current sources is defined by

$$A_{\text{INL}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (19)$$

Each incident matrix element is 1 if the current is arriving to the bus or −1 if the current leaves the respective bus; otherwise it is equal to zero. The dimension of $A_{\text{INL}}$ is equal to the number of connected current sources (2) by the number of buses (3).

The system resonant frequencies can be found by plotting each element of $H_{\text{model}}$ as a function of the frequency (Figure 2). The system transfer matrix, $H_{\text{model}}$, in this case is equal to

$$H_{\text{model}} = Y_{\text{bus}}^{-1} \cdot A_{\text{INL}}. \quad (20)$$

and (21) at the bottom of the page; that is:

$$H_{\text{model}} = \begin{bmatrix} 0.0116 - j0.7785 & 0.0095 + j0.1509 \\ 0.017 - j1.5761 & 0.0114 - j0.7407 \\ 0.0114 - j0.7407 & 0.0271 + j2.2359 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}. \quad (22)$$

The magnitudes of the system current transfer matrix elements as a function of the frequency are shown in Figure 2. Each element of the current transfer matrix, $H_{\text{model}}$, $b_{ij}$, represents the magnitude of the induced voltage in Bus $i$ when 1 A is injected in Bus $j$. The frequency response of each bus depends on the power distribution topology and the associated impedance values. The connected nonlinear loads represented by current sources do not modify the power distribution frequency response.

The system voltage sensitive analysis allows finding the point of connection of the active power filter that produces the best voltage compensation in Bus #1. This can be performed by simulating the connection of a shunt active power filter in each bus of the power distribution system, and calculating the associated voltage distortion in each bus. In this case the incident matrix is equal to

$$A_{\text{INL}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (23)$$

The 1 in the first row presents the active power filter when it is connected in Bus #1, the 1 in the second row represents the active power filter when it is connected in Bus #2, while the 1 in row 3 represents the active power filter when it is connected in Bus #3. By multiplying the
system impedance matrix \( Z_{bus} \) by the incident matrix \( A_{INL} \), the system transfer matrix \( H_{node} \) is obtained. By plotting the absolute values of each transfer matrix element as a function of the frequency the voltage sensitivity analysis is derived (Figure 3).

In particular, Figure 3(a) shows the voltage response in Bus #1, when the active power filter is connected in this bus. This figure shows that if 1 A with a frequency of 1,136 Hz is injected in Bus #1, a 20 per-unit voltage is generated in the same bus. Figure 3(b) shows that if 1 A is injected in Bus #2 with the same frequency, the voltage induced in Bus #1 is 40 per unit, and finally, if 1 A is injected in Bus #3, with the same frequency, the voltage induced in Bus #1 is only 19 per unit [Figure 3(c)].

The voltage sensitivity analysis for Bus #1 shown in Figure 3 concludes that the effectiveness of active compensation in the Bus Voltage #1 is significantly improved if the active power filter is connected in Bus #2, instead of connecting it in Bus #1 or Bus #3. In other words, by injecting current harmonics in Bus #2, the voltage distortion in Bus #1 is reduced more effectively than if the compensation is done in Bus #1 or in Bus #3. This result is correct since the Bus #2 injects more current harmonics to the power distribution system than the nonlinear load connected in Bus #3.

It was shown that with this analysis, the influence of active compensation in all the power distribution buses can be obtained. The selected shunt active power filter point of connection will improve not only the compensation effectiveness of the distribution system, but it will also reduce the required active filter rated power to obtain the same compensation performance.

**Active Shunt Compensation in a Multibus Industrial Power System**

Figure 4 shows the single line diagram of an industrial power distribution system composed of large power nonlinear loads, motors, and passive filters. The THD in current and voltages at the point of common coupling (Bus #1) exceeds the maximum values recommended by ANSI/IEEE Standard 519-1992. For this reason, it is necessary to connect a shunt active power filter that will help to reduce the THD in current and voltage at the PCC (Bus #1). From the simple observation of the power distribution system shown in Figure 4, the point of connection of the shunt active power filter cannot be derived. Moreover, the compensation effectiveness of the shunt active power filter can be severely affected if this compensator is not connected in the proper bus. By using the analytical procedure developed earlier, the power distribution operating characteristics are derived, so that the best connecting point of the shunt active power filter can be obtained.

**Resonant Frequencies**

The resonant frequencies for different buses will be obtained (from the analysis of matrix \( H_{node} \)), and the voltage sensitivity of Bus #1 will be evaluated. Resonant frequencies activated by current injection in Bus #4 in different power distribution buses are shown in Figure 5. It is clear that current injection in Bus #4 generates resonant frequencies at 1,486 and 578 Hz. The voltage waveforms in Buses 1–3, 6, and 7 are more affected when the resonant frequency is equal to 578 Hz. A more complete analysis proves that, in this case, resonances do not increase voltages in the power distribution buses, making the system more reliable.

**Voltage Sensitivity Analysis**

This analysis allows finding the effect of current injection in different system buses and the voltage that is induced by these currents in a particular bus of the power distribution system. This result is very important to determine the best point of connection of an active power filter that will keep the voltage THD in Bus #1 below 3%. Moreover, once the best point of connection is defined, it is possible to evaluate the effect of the active compensation in all the other power system buses.
The result of this analysis is shown in Figure 6 and illustrates the magnitude of each element of row 1 of the current transfer matrix $H_{node1}$. This figure shows that by connecting the shunt active power filter in Bus #4 the voltage distortion in all the other buses is reduced, and the magnitude of the compensating current that is required to inject to the power distribution system is smaller.

![Power distribution system impedance frequency response](image)

- (a) Frequency response for Bus #1.
- (b) Frequency response for Bus #2.
- (c) Frequency response for Bus #3.

![Voltage sensitivity analysis](image)

- (a) Voltage response if the current is injected in Bus #1.
- (b) Voltage response if the current is injected in Bus #2.
- (c) Voltage response if the current is injected in Bus #3.
Table 1 shows the current amplitudes that need to be injected in each bus to have a voltage distortion equal to zero in Bus #1. It is clear that by connecting the active power filter in Bus #4, the rms value of the active power filter current is reduced (1,144 A). The first column indicates the active power filter point of connection.

Also, Table 1 shows that if the active power filter is connected in Bus #1, a larger rms compensating current is required to keep the THDv in Bus #1 below 3%, and a larger amplitude of current harmonic components is required to keep the low voltage distortion. A similar effect is obtained if the active power filter is connected in Buses 2, 3, 5, 6, or 7. This result proves that the compensation effectiveness of the shunt active power filter depends on the point of connection.

Table 2 shows the effect in voltage distortion in the distribution system for different points of connection of the active power filter. Again, the best point of connection is Bus #4, since by injecting the active power filter current in this bus, the voltage distortion in all the power system buses is reduced.

Table 2 also shows that even though the connection of the active power filter in Buses 1, 2 or 3, reduces the associated THDv, it significantly increases the voltage distortion in Buses 4, 5, and 6, due to resonances generated by the same active filter. Again, this result confirms the fact that compensation effectiveness in a multibus power system can be effectively improved by the proper selection of the active power filter point of connection.
Conclusion
A numerical analysis based on an analytical procedure that determines the more effective point of connection of shunt active power filters in multibus power distribution systems has been proposed. With this algorithm active current harmonics compensation performance can be optimized by reducing voltage distortion in the power distribution and requiring a smaller compensation current. A procedure based in the power distribution system transfer function matrices has been derived and carefully explained in a multibus industrial power distribution steel mill plant.

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References

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