A WIDE RANGE SENSORLESS SPEED ESTIMATOR BASED ON ROTOR IRREGULARITIES FOR INDUCTION MOTORS

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Abstract.- This work presents a method of speed estimation for induction motors, based on the detection of side-band frequencies which are produced by dynamic eccentricities of the rotor. The method uses two different ways to measure speeds: one for speeds lower than 600 rpm and other for higher speeds. In the first case, a high frequency carrier signal is introduced, and in the second case, the normal frequency current of the inverter is used. In both the cases the stator current signal is filtered and then self-multiplied. The rotor irregularities will produce in this current signal, amplitude modulations proportional to rotor speed and independent of the machine parameters. This information is carefully isolated through the help of programmable filters and high precision comparators to get the rotor speed. Computer simulations and some experiments have been carried out, and they have shown that it is possible to measure speeds as low as 20 rpm.

1. INTRODUCTION

The recent advances in power electronics have permitted the implementation of sophisticated methods of control for induction machines. Most of these methods require a precise estimation of rotor speed. Traditionally, mechanical tachometers have been used, but they have the disadvantage of having mechanical problems and maintenance requirements. Recently, numerous attempts to estimate the speed machine without tachometer are being proposed [1-3]. Most of them are based on the knowledge of the parameters of the machine from where the flux, torque or speed can be evaluated to control the motor. With the parameters, the speed can be estimated through the slip information hidden in the equivalent impedance of the machine. However, the parameters of the machine are not constant because they depend strongly on machine temperature and saturation of the magnetic circuit. In recent years, methods based on small rotor irregularities have been reported [4, 5]. These methods are independent of the parameters but, with the techniques used until now, speeds lower than 100 rpm have not been reported.

In this work, a method able to measure speeds as low as 20 rpm has been developed. The method is based on the self multiplication of the induction machine current signal, and uses two different forms to measure speed: one for speeds lower than 600 rpm through the addition of a high frequency carrier, and other for higher speeds where the normal frequency currents of the stator are used. In the first case, the high frequency carrier introduced in the stator currents (400 Hz), is sensed and isolated from the normal inverter frequency, and then multiplied by itself. In the second case, the stator current is directly filtered with the help of frequency programmable filters and also multiplied by itself. In both the cases, the self-multiplication and amplification of specific side-band frequencies, which are produced by rotor irregularities, are used to estimate the speed.

The irregularities of the rotor considered in this work are the eccentricities of the shaft with respect to both, rotor and stator. Fig. 1 shows a schematic of these eccentricities.
When only exists eccentricity of the shaft with respect to stator, no side-band frequencies are generated and hence no speed information is obtainable. When the eccentricity is only with respect to rotor, the rotation of the shaft will produce a reactance variation each time the rotor passes through a pole. For example, in a four pole machine, one complete rotation will produce four amplitude pulsations in the machine currents, which represent a particular side-band. Now, when both, rotor and stator have eccentricities, which is the normal case encountered in experiments, a second couple of side-band frequencies, which produces one pulsation per rotor turn (independent of the number of poles), superimposed to the first one, is generated. As it will be explained later, they correspond to dynamic eccentricities of order 4 and 1 respectively ($n_e=4$ and $n_e=1$).

II. LOW SPEEDS ESTIMATION

As it was already mentioned, the speed estimation for low speeds is realized adding to the output frequency inverter, a fixed frequency carrier signal, whose rotating field is filtered during detection stages, to avoid interaction with other inverter frequencies. As this method is used for low speeds only, that means, until around 20 Hz (600 rpm for a four-pole machine), the carrier signal has to be bigger than 20 Hz. On the other hand, this signal neither must interfere with the PWM pulsations. Assuming in 2 kHz the minimum frequency of the PWM, the carrier signal has to be set in a value bigger than 20 Hz and lower than 2 kHz. 400 Hz has been choosen for this carrier signal. In this form, the carrier frequency and their side-bands generated, can be easily filtered from the normal operating frequencies of the inverter, isolated and processed for speed estimation.

According with references [4] and [6], the side-band frequencies can be evaluated and related with the rotor speed through the following equations:

$$f_{shi} = f_i \left( nR \pm n_e \left( \frac{1-s}{p} \right) \right) \pm n_{WS}$$  \hspace{1cm} (1)

$$\Delta f = f_{shi}^+ - f_i = f_i - f_{shi}^-$$  \hspace{1cm} (2)

$$\omega_m = k \cdot \Delta f$$  \hspace{1cm} (3)

where:

- $i$ = 1 (main frequency); 2 (carrier frequency)
- $f_{shi}$ : side-band frequency generated
- $R$ : number of rotor slots
- $n$ : any integer = 0, 1, 2, ...
- $n_{WS}$ : order number of stator mmf time harmonic
- $n_e$ : dynamic eccentricity order = 1, 2, ...
- $f_i$ : fundamental frequency
- $S_i$ : slip relative to $f_i$
- $p$ : number of pair of poles
- $k$ : constant which depends on $n_e$

As it was already mentioned, in a four-pole machine, the eccentricity orders $n_e=1$ and $n_e=4$ are produced by the stator and rotor respectively. Due to the fact that the speeds to be measured are in the low speed range, it is better to isolate $n_e=4$, because four pulses for each rotor turn will be obtained. Besides, the side-band generated is four times farer from the carrier than the ones who correspond to $n_e=1$. This fact makes the side-bands obtained with $n_e=4$ easier to isolate. However, the
distance between the carrier and the side-band frequencies can still be less than 2 Hz, and also, because eccentricities are normally very small, the side-band amplitudes are small compared with the carrier. This situation makes quite difficult to separate the side-band frequencies from the carrier, using simple filters.

In this work, the isolation of the side-bands from the carrier, is based on the self-multiplication of the carrier signal, to obtain the difference frequency, Δf, related with \( n_e = 4 \). To get the speed of the rotor in rpm, Δf is multiplied by 60 and then divided by the value of \( n_e = 4 \), that means, by the constant \( k = 15 \). Fig. 2 shows the frequency spectrum produced by the 400 Hz carrier. It can be seen that the left side-band which corresponds to \( n_e = 4 \) is bigger and more separated from the carrier than the one who corresponds to \( n_e = 1 \).

**Fig. 2. Spectrum of the frequency carrier**

When the speed becomes higher than 600 rpm (four-pole machine), the carrier and its side-bands becomes closer to the fundamental frequency of the inverter. This situation complicates the separation and amplification of the carrier from the relatively high amplitude of the fundamental frequency of the inverter. For this reason, a different method to estimate high speeds has been implemented.

### III. HIGH SPEED ESTIMATION

In the high speed region (600 to 3000 rpm for a four-pole machine), the carrier signal is eliminated and the normal frequency of the inverter is used to detect the speed. In this case, the side-band \( n_e = 4 \) closely approaches the fundamental, as shown in Fig. 3 and then is not easy to isolate. For this reason, the side-band who corresponds to \( n_e = 1 \) is used.

**Fig. 3. Normal frequency spectrum**

Thus, the main stator current signal is directly self-multiplied and the envelope of the multiplier signal is isolated through low pass programmable filters. These filters are able to change their cut-off frequency by means of an external clock signal which is synchronized with the fundamental frequency of the inverter. The aforementioned envelope of the multiplier signal, carries the information of the difference between the fundamental and its left side-band, Δf, which is proportional to rotor speed. After that, by multiplying Δf by the constant k, whose value is 60 for \( n_e = 1 \), the speed in rpm is obtained.

### IV. CIRCUITS DESIGN FOR SPEED DETECTION

The voltage amplitude of the carrier signal, used for low speed evaluation, has been set in 15% of the nominal value of the fundamental. In this form, parasitic torque are negligible. This also
makes the side-band frequencies extremely small. As an example, the Fig. 4 shows the frequency spectrum of the stator current obtained when a four-pole machine is fed with a fundamental of 5 Hz and rotates a 147 rpm. The fundamental and the 400 Hz carrier, with their respective side-band frequencies are also displayed. As it can be seen in this figure, the amplitude of the side-band who contains the speed information is very low compared with the carrier.

![Frequency spectrum of the stator signal](image)

*Fig. 4. Frequency spectrum of the stator signal*

The first step is then to increase the amplitude of the carrier signal with respect to the main frequency of the inverter. The carrier signal is filtered and amplified with respect to the main frequency, and then multiplied by itself. This multiplication allows the separation of the main frequencies from the side-band frequencies through the sum, difference and double frequencies which appear in this operation. After the multiplication of the signal, another filtering through a low-pass filter is required from where the frequency spectrum shown in Fig. 5 is obtained. In the process, the highest amplitude corresponds to a difference frequency, $\Delta f$, which is the basis of the speed evaluation.

The Fig. 5. b) shows the waveform generated with the spectrum of fig. 5 a). It can be noted that, despite the large amount of frequencies contained in this waveform, it looks almost sinusoidal, being $\Delta f$ its main frequency, due to its high relative magnitude. Then, the speed can be computed directly from the waveform of Fig. 5. b), with the help of an electronic counter, which is implemented with a microprocessor.

![Fig. 5](image)

*Fig. 5. a) Frequency spectrum after multiplication and filtering. b) Waveform generated with the spectrum of a)*

When the speed is higher than 600 rpm, the microprocessor eliminates the 400 Hz carrier signal and processes directly the fundamental frequency of the inverter. After that, the first step is the elimination of the highest harmonic components through a low pass filter, with a 200 Hz cut-off frequency. Then, the signal is selfmultiplied and filtered with low pass, programmable filters. These filters change their cut-off frequency using an external clock obtained from the microprocessor. The clock is also related with the fundamental frequency of the inverter.

The Fig. 6 shows a functional block diagram of the overall system described, which comprises a high pass filter to increase the magnitude of the carrier with respect to the main inverter frequency, and an analog multiplier. From the output of the multiplier, the envelope of the carrier, which corresponds to $\Delta f$, is isolated through a low pass filter, which has a fixed cut-off
frequency for the low speed method and a programmable cut-off frequency for the high speed method. Then, this filtered envelope signal is compared with its dc component to get a square waveform whose frequency is $\Delta f$.

V. EXPERIMENTAL RESULTS

Experiments were carried out using four different induction machines: i) a 2HP, 4 pole, 220 V, 50 Hz, ii) a 2 HP, 4 pole, 380 V, 50 Hz, iii) a 5 HP, 4 pole, 380 V, 50 Hz and iv) a 12 Hp, 6 pole, 380 V, 50 Hz. From all of them, the speed estimation using the proposed methods was successful. In this experiments, it was found that the method based on the detection of side-band frequencies can measure lower speeds than expected, due to a good separation between $\Delta f$ and the others frequencies.

The Fig. 7a) shows the current waveform of the frequency carrier and Fig. 7b) shows the output signal obtained after the carrier is multiplied by itself. The envelope which contains the information of $\Delta f$ has been marked with circles in 7b), allowing to see almost one period of this frequency.

![Fig. 7. a) Frequency carrier waveform. b) Self-multiplication of the carrier](image)

The signal obtained from Fig. 7b) is filtered to get the envelope $\Delta f$, which is shown in Fig. 8. The frequency of this envelope, according with eq. (3), corresponds to 250 rpm.

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Subsequently, this envelope is compared with a constant signal to get a square waveform which is proportional to the machine speed. The Fig. 9 shows now the envelope $\Delta f$ with its square waveform, for a speed of 19 rpm. At this speed, the waveform is not so clear but the speed is perfectly recognized. In this figure it is also possible to see another frequency superimposed over $\Delta f$, which correspond to a frequency generated by rotor slots. With a good filtering system, this new frequency could be isolated, allowing to measure even lower speeds.

![Fig. 8. Frequency envelope for 250 rpm](image)

![Fig. 9. Speed signal for 19 rpm.](image)

*a) frequency envelope. (\(\Delta f\)) b) associated square waveform.*

An experimental result using the method for high speed estimation is shown in Fig. 10. In this case, a speed of 622 rpm with 24.4 Hz of main frequency has been used. In 10a), the multiplied signal waveform is displayed and in 10b), the envelope obtained from a) is presented. The envelope frequency, which correspond to $\Delta f=10.36$ Hz, is related with the speed, through eq. (3), and using $n_e=1$.

<table>
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<tr>
<th>Freq. Inv. (Hz)</th>
<th>Speed (rpm)</th>
<th>$\Delta f$ (Hz)</th>
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<tr>
<td>14.5</td>
<td>355</td>
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*Table 1. Values of $\Delta f$ for the high speed method.*

It can be also mentioned that with the 12 HP, 6 poles machine tested, a speed of 2 rpm was computed with the circuit. This was possible because the harmonics generated by the rotor slots were large enough in amplitude to be measured and they produced 40 periods of $\Delta f$ ($n_e=40$) with just one turn of the rotor.
VI. CONCLUSIONS

A different method to detect the speed of induction motors have been proposed. The method is based on the addition of a fixed carrier frequency signal to the PWM modulation, to measure low speeds. This carrier permits the detection of side-band frequencies generated by a fourth-order rotor eccentricity, which are substracted from the carrier to obtain $\Delta f$. This frequency is multiplied by a constat to get the speed of the machine in rpm with high precision.

To detect medium and high speeds, the normal current signal of the stator is directly used, and the high frequency carrier is eliminated. This signal is self-multiplied and the envelope, which correponds to a first-order rotor eccentricity, is obtained through programmable low pass filters. In this form, the side-band frequency oscillation, which contains the exact information of the rotor speed, is isolated and transformed to a square-wave whose frequency is proportional to machine speed.

Both methods are independent of the electromagnetic parameters because the induced harmonics are based on the mechanical eccentricities of the rotor. It has also been verified that with some kind of induction motors, high order side-band harmonics, produced by rotor slots, can be detected, making it possible to measure speeds as low as 2 rpm. However, with the motors tested in laboratory, this was feasible only in the 12 HP, 6 poles machine. Despite it, 20 rpm as a lowest speed limit for the other machines was possible by isolating the fourth-order side-band frequency.

It has also been found that the method is sensitive to noisy environments, because spurious signals can produce counting errors in the microprocessor. To avoid it, careful isolation of the speed signals must be implemented.

The speed detection method proposed in this paper, can be improved by intentionally intruducing irregularities in the rotor. These irregularities have to be small enough to avoid parasitic torques and slot harmonics generation. If the number of irregularities introduced in the rotor are multiple of the number of poles of the machine, higher amplitude side-band frequencies than the normal existing in the motor, can be generated. The order of these side-band frequencies, will depend on the number of eccentricities introduced with respect to the poles of the machine.

Computer simulations and experiments with different induction machines have permitted to verify the feasibility of practical implementation.

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REFERENCES


