Approaches to transmission planning: a transmission expansion game

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Abstract—Defining and making decisions about network investments has become a hard task in a competitive environment. This work defines a methodology to look for expansion alternatives in a transmission system. We propose a Transmission Expansion Game model that consists of four main elements: i) generating transmission expansion plan scenarios, ii) valuation of a project based on the design of a linear contract, bargaining solutions and hidden actions, iii) optimal value of a risky investor’s portfolio made up of several projects, and iv) transmission cost allocation with wind energy assets.

The results obtained show that the model developed is efficient to solve the combinatorial problem. A principal-agent model obtains the real costs of the bidders and creates incentives for disclosure of information. The private optimal portfolio and bargaining for allocation cost gives evidence for the central planner to adjust the project assignment process and to carry out the proposed expansion plan efficiently. To test the methodology we analyze the Chilean Central Interconnected System.

Index Terms—Transmission expansion game, Principal-Agent model, incentives, game theory, Conditional Value at Risk (CVaR), investment, bidding contract, ordinal, meta-heuristic, multi-objective optimization.

I. INTRODUCTION

The agents participating in a Transmission Expansion Plan (TEP) are rational, and, as such, they maximize their utilities. The decision-making process associated to the TEP must consider both cooperative behavior as well as non-cooperative to decide the expansion required by the system.

In a centralized model, the planner defines the optimal expansion plan. The planner expects that the expansion plan is allocated to the investors with efficient bids. However, an efficient investor is faced with various risks that influence his decision-making process and to carry out the proposed expansion plan efficiently. To test the methodology we analyze the Chilean Central Interconnected System.

I. INTRODUCTION

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In a centralized model, the planner defines the optimal expansion plan. The planner expects that the expansion plan is allocated to the investors with efficient bids. However, an efficient investor is faced with various risks that influence his bids or the selection of projects to bid. In the last decades, rejection to construct a transmission line and land cost overruns have increased. This has resulted in inevitable delays. Therefore, the transmission regulator should explore incentives to encourage investment and mitigate overhead.

Investment in transmission networks is characterized by the following: long periods of construction, high capital requirements, long service life of the assets, and long periods of payback. In [1][2][3], the most relevant aspects in transmission expansion are described. The basic criteria considered in the TEP have been: improvement of reliability, increase in the availability of supply, and fostering of competition among electricity market agents.

To solve the TEP problem, the algorithms are classified according to the solution technique, usually mathematical programming and heuristic techniques [1][4][5]. Two types of scenarios are used; static and dynamic scenarios, the former one being the most used. The most recent applications have applied multi-period planning with evolutionary algorithms. Hybrid applications such as Ordinal Optimization (OO) and Tabu Search (TS) or Genetic Algorithms (GA) with the consideration of social benefit functions and multi-objective algorithms have been used [6].

Several economic models, mainly oligopolistic, such as Cournot, Bertrand and Stackelberg, are usually considered. These models represent the behavior of players and game dynamics (strategies and payments) [7]. The financial valuation depends on the models, on who the owners are and who are the ones that control transmission assets [6]. The most popular methods to evaluate transmission are the net present value and the cost/benefit analysis. The most common techniques are binomial trees, real options, Markowitz’s portfolio theory and risk analysis [8][9].

Additionally, there is several techniques to define the use of the network based on either physical or economic laws. From an economic perspective, there are two network usage approaches: one as a function of the capacity and the other as a function of the energy. In general, game theory has been the most commonly used tool to allocate costs in network expansions. Shapley value, bilateral Shapley value, Kernel, and nucleolus, have been commonly used.

The main contribution of this paper is to use different techniques and to identify feasible solutions to the TEP, proposing four approaches: formulation of the TEP combinatorial problem, estimation of the cost and the results of bidding competition, investment risk formulation, and allocation of the transmission cost. We consider ordinal multi-objective optimization. This optimization allows us to reduce the search of possible solutions as well as a trade-off between investment and operation cost, energy diversity and CO₂ emissions. The developed model determines the bargaining solutions where uncertainty creates cost overruns and may affect the feasibility of the optimal expansion plan. In addition, it determines the relationship between efficiency in the assignment of projects and the number and type of investors who participate in the auction process. This leads to incentives that encourage investment. We analyze the
interaction between the allocation cost method and economic rationality.

Section II proposes a transmission expansion search methodology. The techniques implemented consider \(OO\), multi-objective optimization, \(TS\) with path re-linking and reliability criterion. Section III formulates the three main elements of project valuation: Optimal cost, efficient bidding and investment risk. We present bargaining solutions, the Principal-Agent model, incentives in the design of contracts and a private optimal portfolio using \(CyRA\). In Section IV, we show topics on mechanism design, incentives and allocation cost method, and a hybrid cost allocation method is proposed. Section V presents a case study: the 55-bus reduced Chilean Central Interconnected System model. Finally, conclusions are drawn in Section VI.

II. TRANSMISSION EXPANSION PLAN - TEP

The main objective of grid investment is to minimize the investment and/or operating costs of the system subject to economic, reliability and sustainability constraints. This section has three main objectives: i) reduction of the search space through the \(OO\) method, ii) multi-objective optimization under the concept of Pareto dominance, and iii) intelligent solutions search by means of \(TS\) and path re-linking with \(N-1\) criterion [6].

A. Reducing the search space and system cost

The first phase starts with the definition of the number of \(TEP\) samples to be considered in the \(OO\) [10]. The number of samples depends on the number of possible transmission assets considering technical and economic information. However, the feasible solutions of this subset are not known with accuracy, so a subset of \(S\) solutions is established, which allows to infer how many samples of the solution space are enough to find an estimated optimal to be found in \(G\) and \(S\). The accumulated probability function of the hyper-geometric distribution and investment cost restrictions and the maximum lines to expand by expansion plan are used to define the number of samples. Now, for each expansion plan, the balance of the energy market is established. An optimal \(DC\) power flow is used, which determines the minimum operation cost. Only the production costs \((PC)\) of the generators of the system are considered. In turn, the congestion cost \((CC)\) is established, which is defined as the difference between the base cost of the system and the one of the system after expansion. We calculate the investment annuity and operation cost for each \(TEP\). In this manner, the net social welfare (sum of surpluses) maximization generated for each expansion plan \(k\) is attempted (1).

\[
CC_k = \min PC_k - \min PC_{base}
\]

B. Multi-objective and Pareto plans

A concept that is commonly applied in multi-objective optimization is Pareto dominance. The solution to the multi-objective problem is a solution set or a solution vector that is not dominated by another vector. Three objectives are considered to identify a list of Pareto solutions. The objectives are: annual cost (investment and operation), equivalent \(CO_2\) emissions in tCO2e/MW-h, and the technological mix diversity. Diversity, \(I_k\), is defined as a supply security mechanism [11] (2).

\[
I_k = \sum p_i \ln(p_i)
\]

where \(p_i\) is the share of each technology in the economic dispatch. It must be noted that the objective is maximization, that is, the intent is to maximize the system’s diversity. The equal weighted aggregation method, by means of the linear combination of the objectives, is considered. Therefore, to evaluate the objective functions under the same criterion, the minimum distance existing between the maximum diversity and the annual average diversity of the economic dispatch are considered.

C. Seeking the optimal and reliability criterion

This optimization phase considers the \(TS\) meta-heuristics developed by Glover [12]. The enhancement strategy is related to flow sensitivity and the diversification strategy to path re-linking. \(Path\ re-linking\) finds potentially attractive configurations from the Elite-Pareto configurations, and some of the attributes existing in the Elite-Pareto configuration or solution that are not contained in path re-linking are copied.

The \(TS\) process starts with the minimum cost solution found in the \(OO\). The neighborhood is generated, the sensitivity criterion is used with respect to the level of utilization and congestion of the system’s lines and the addition or removal is made depending on which is the case for each neighbor. Pareto solutions are obtained for each neighborhood, which are stored in a Pareto list and a minimum cost solution is stored in the Tabu list. A Pareto solution to be used in the next iteration is chosen, a new neighborhood is created and the process is repeated. In turn, the path re-linking process is considered with respect to the Pareto solutions found in the \(OO\) and multi-objective optimizations. The amount of neighbors was defined proportional to the number of possible expansions and the stop criterion with the \(OO\) methodology.

D. Reliability criterion: \(N-1\) / Failure Cost

The neighborhood solutions in the \(TS\) are evaluated against the \(N-1\) reliability criterion. The reliability criterion considers the restricted \(N-1\) criterion when comparing the annuity of the investment cost with the failure cost and the non-supplied energy cost (a concept used in Chile), so it is possible to consider solutions with load shedding different from zero. In addition, a simple contingency is considered in the transmission lines to identify the maximum load shedding in case of a failure. A \(minmax\) criterion is used to find the optimal solution. The maximum load shedding is identified together with the worst condition for the \(N-1\) criterion for each \(TEP\) generated in neighborhood \(Z_{N-1}\), and, by means of the Euclidian distance, the minimum load shedding with the annuity of the associated \(X\ TEP\) is chosen (3).

\[
\max_{X} \min_{Z_{N-1}} Plan_{k,p}(X,Z_{N-1})
\]

where \(Plan_{k,p}\) represents the expansion plan for neighborhood \(V\) for each iteration \(p\). Finally, the optimal \(TEP\) is obtained.

III. COST, BIDS AND INVESTMENT IN TEP

The valuation of a project is defined in terms of the costs and expected bid that maximize its utility. This section has
three main objectives: i) the valuation of transmission expansion projects with hidden information and overrun cost, ii) the bargaining mechanism of the right-of-way costs, and iii) bidding competition assuming investment risk.

A. Overrun cost and Bargain problems

The optimal costs and activities will depend on how effective is the one who performs them. We consider that the expected cost of project \( j \in \text{TEP}_n \) for agent \( n \) is a function of the initial asset cost, \( C_j^n \), the bargaining solution of the Right-of-Way cost, \( \text{RoW}_j^n \), and the level of effort, \( e_j^n \) (4):

\[
C_j^n = C_j^n + \text{RoW}_j^n + \phi_j^n - e_j^n
\]

with an expected minimum cost, \( C_j^n \), and an expected maximum cost, \( C_j^n \). \( \phi_j^n \) is a random variable that represents unpredictable costs, and \( e_j^n \) is the cost reduction due to the effort of the investor (Section III-B). Note that \( C_j^n \) is only known to the agent as a distribution function, \( G(C_j^n) \). \( F(\phi_j^n) \) is the cost overrun distribution function of project \( j \), and the expected value of \( \phi_j^n \) follows a normal distribution \( N[0, \sigma_j^n] \) [13]. To determine the \( \text{RoW}_j^n \) we assume a bargaining game between the land owner and the investor (on one side, the investor wants to pay as little as possible, but, on the other side, the land owner want to receive as much as possible, even to make them delay or forbid the construction of a project in order to make it extremely profitable). The solution to these problems was proposed by Nash and is known as the Nash bargaining solution [14]. The bilateral negotiation problem consists of a pair \( (S, d) \), where \( S \) is a subset of \( \mathbb{R}^n \) and \( d \in S \). The set \( S \) represents the set of pairs of payments when the players act cooperatively and \( d \) is the payment pair when they act non-cooperatively, that is, this is the payment when there is no agreement. The set \( S \) is convex and has finite alternatives with a payment \( x \) such that \( x \geq d \), for all \( x \in S \), and \( x > d \) for some \( x \in S \). The Nash bargaining solution, \( x^* \), is the outcome of \( (S, d) \) for which the utility product \( (x_1 - d_1) \cdots (x_n - d_n) \) is maximum, hence (5) holds:

\[
x^* = \text{arg max}_{\text{RoW}} \max_{x \in S} \max_{d \in S} (x_1 - d_1) \cdots (x_n - d_n)
\]

where \( d \) is defined as the utility of disagreement point. Here, the independence to irrelevant alternatives axiom does not necessarily hold, since there may be an alternative that is not feasible. In addition, the Nash bargaining solution does not meet the individual monotonicity property. This property was proposed by Kalai-Smorodinsky, where the solution \( (S, d) \) is determined by the intersection of the Pareto frontier in \( S \) and the line joining the disagreement point with the utopia point. The utopia point, \( u_p \), is the maximum utility that each player wishes to achieve, \( u(\text{RoW}) = \max \{ \text{RoW} \in x \} \). The Kalai-Smorodinsky solution [14], \( x_K^* \), is defined in (6):

\[
x_K^*(S, d) = \text{arg min}_{x \in S} \max_{d \in S} (x_1 - d_1) \cdots (x_n - d_n)
\]

such that \( (x_1, x_2) \in S \) and (7) holds.

\[
\max_{x \in S} \rightarrow \frac{x_1 - d_1}{d_1} = \frac{x_2 - d_2}{d_2}
\]

B. Hidden and effort actions

There are two types of information problems: hidden actions (moral hazard) and hidden information (adverse selection) [14]. The Principal-Agent model can be stated as a game where the principal central planner/regulator delegates decisions to an agent investor.

Considering that there are \( n \) agents and \( j \) projects, the profit function, \( \pi_j^n \), can be quantified and is determined by the effort level \( \lambda_j^n e_j^n \), where \( \lambda_j^n \) is the coefficient of the level of effort which reflects the effort level on performance of agent \( n \) and project \( j \). The profit is defined by (8):

\[
\pi_j^n = \lambda_j^n e_j^n + w_j^n
\]

where \( w_j^n \) is a random variable, \( w_j^n = N(0, \sigma_j^n) \), thus the expected value of the level of effort is \( \mathbb{E}[\pi_j^n] = \lambda_j^n e_j^n \) and the variance of \( \pi_j^n \) is \( \sigma_j^n \). The principal decides the payment function \( T_j^n \) for the agent. The function is assumed linear and expresses the monetary transfer to the agent such that it \( T_j^n = \lambda_j^n + \xi_j^n \cdot \pi_j^n \), where \( \lambda_j^n \) is a fixed transfer and \( \xi_j^n \) is the degree of risk assumed by the agent (if he does not assume any risk, \( \xi_j^n = 0 \) and if he assumes all the risk, \( \xi_j^n = 1 \)). The effort cost function \( g_j^n \) is a quadratic function as \( g_j^n = 0.5 \cdot \gamma_j^n \cdot (e_j^n)^2 \), where \( \gamma_j^n \) is the positive cost coefficient of effort. The expected utility of the principal, \( U_{\pi_p}^j \), is:

\[
\mathbb{E}[U_{\pi_p}^j] = \pi_j^n - T_j^n = \lambda_j^n e_j^n + w_j^n - [\lambda_j^n + \xi_j^n \cdot \pi_j^n]
\]

and the expected income, \( I_j^n \), of the agent is:

\[
\mathbb{E}[I_j^n] = T_j^n - g_j^n = \lambda_j^n + \xi_j^n \cdot \pi_j^n - 0.5 \cdot \gamma_j^n \cdot (e_j^n)^2
\]

It is assumed that the agent is risk-averse which means that the certainty equivalent income equals the mean of the random income subtracting the risk cost. The risk cost criteria defines that the risk cost is \( 0.5 \cdot \gamma_j^n \cdot \pi_j^n \cdot (\sigma_j^n)^2 \), where \( \rho_j^n \) is the degree of risk aversion. We define \( I_j^n \) in (11):

\[
\mathbb{E}[I_j^n] = \lambda_j^n + \xi_j^n \cdot \pi_j^n - 0.5 \cdot \gamma_j^n \cdot (e_j^n)^2 - 0.5 \cdot \gamma_j^n \cdot \rho_j^n \cdot (\sigma_j^n)^2
\]

Considering the Individual Rationality constraint, IR, the agent participates if \( \mathbb{E}[I_j^n] \geq z_j^n \), where \( z_j^n \) is the minimum profit or income to reinvest in the project, and the incentive compatibility constraint, IC, makes the agent to choose the effort level \( e_j^n \) to maximize \( I_j^n \) such that \( e_j^n \) is equal to \( \arg \max \{ T_j^n - g_j^n \} \). Applying the first order condition we establish that \( \delta I_j^n / \delta e_j^n \geq 0 \) [14]. Then, the maximization problem is:

\[
\max_{x_j^n, e_j^n} \mathbb{E}[U_{\pi_p}^j] = \mathbb{E}[U_{\pi_p}^j] = (1 - \xi_j^n) \cdot \lambda_j^n \cdot e_j^n - \kappa_j^n
\]

subject to

\[
(\text{IR}) \quad \mathbb{E}[I_j^n] \geq z_j^n
\]

\[
(\text{IC}) \quad \frac{\delta I_j^n}{\delta e_j^n} \geq 0
\]

C. Bidding competition and efficient bid

A contract is commonly used to define the relationship between a central planner/regulator and an investor. Competing and bidding for contracts means that the central planner (users) expects to pay the minimum possible for the contract. If we consider a contract of the linear type so that the value of project \( j \), \( V_j \), is a linear combination between the bid and the cost [13], we get:

\[
V_j = a_j C_j^n + b_j h_j^n + \chi_j
\]

where \( b_j^n \) is the bid for project \( j \), and \( a_j \), \( b_j \), \( \chi_j \) are constant values. Bid \( b_j^n \) may be done in a single-valued auction or in a first-price sealed envelope auction. The latter type has no dominant equilibrium, however, it satisfies a (weak) Nash
equilibrium. Each bidder chooses his best bid guessing about the decision rules followed by other bidders. Thus, if bidder \(i\) has a valuation or optimal cost, \(v_i\), with the \(b_i\) bid belonging to the supply function \(B\) (monotonically increasing function), the gain will be determined by \((v_i - b_i)\). If \(B(v_i)\) are the bids of the other \(k\) agents, agent \(i\) will win the tender if \(b_i > B(v_i)\) with a probability \([F(B^{-1}(b_i))]^{n_i}\), where \(F\) is the distribution function of the valuations, \(v_i\) and \(n_i\) is the number of agents that participate in the tender. Imposing the rationality expectation condition to the Nash equilibrium the bid function is [13]:

\[
B(v_i) = v_i - \frac{\int_{v_i}^{\infty} F(x)^{n_i} \, dx}{\int_{v_i}^{\infty} F(x)^{n_i-1} \, dx}, \quad i = 1, 2, 3, \ldots, n
\] (14)

If the distribution function of the evaluations is uniform and the minimum valuation is \(v_i = 0\), then we obtain the optimal bid, \(b_i(v)\), determined by (15):

\[
b_i(v) = \frac{v_i \cdot (n_j - 1)}{n_j}
\] (15)

Where \(v_i\) is the optimal cost, \(C_i\), of the winner of the auction. The contract is of the incentive-type [13], that is, \(v_i = (1-a_i)\), where \(a_i\) is the cost-share factor with \(0 < a_i < 1\) and \(v_i = 0\). The value \(V_j\) is the expected payment by the principal, subject to the bid, \(b_i\), that entails the lowest expected value of project \(j\):

\[
V_j = E[(1-a_j) \cdot b_j + a_j \cdot C_j]
\] (16)

**D. Investment Risk**

The optimal bid in a first-price auction (tender with the lowest Net Present Value, total value or annuity) will depend on incentives and project risks. The risks of competition, financial risks and technical risks are elements to consider not only by investors but also by the central planner. We present a method based on CVaR to construct a project portfolio of an investor. The analysis and optimization of the selection of a portfolio with CVaR as risk measure should consider that the density function of the risk factor is feasible. Approximation methods and/or scenario analysis are used to estimate these risk factors. We use the CVaR linear approximation (17) proposed by Uryasev [15]:

\[
CVaR_P = VaR_P - \frac{1}{1 - \omega} \sum_{s} \pi_s \cdot \eta_s
\] (17)

where \(\eta_s\) is a positive auxiliary variable in scenario \(s\), and \(\pi_s\) is its associated probability with a confidence level \(\omega\).

We consider that \(C_j\) is determined by (4) and the tender is assigned to the annuity cost with the lowest value and annuity factor, \(i\), with discount rate, \(r\), and lifetime asset, \(l\):

\[
aC_{j,s} = r \cdot C_j \cdot [1 + \lambda_{j,s}]
\] (18)

where \(\lambda_{j,s}\) represents the operation, maintenance and administration cost as % of \(C_j\). According to (16), \(V_j\) represents the value at which the construction of project \(j\) is assigned. Moreover, considering (15), \(V_j\) is dependent on the number of bidders, \(n_j\), and the cost recognition factor, \(n_j\), of the tender:

\[
V_{j,s} = C_j \cdot \left( n_j + a_j - 1 \right) / n_j
\] (19)

An investor should know the expected value because this value determines the regulated revenue of a project during its lifetime. In our case it is assumed that the benefit of the annual income depends on two principal risks: cost overrun of \(\lambda_{j,s}\) and penalties, \(m_j\), for delays, \(\delta_{j,s}\), in the execution and operation of the project. Based on the previous items, we consider that the annual income function and \(r\) annuity factor is given by:

\[
aV_{j,s} = r \cdot V_{j,s} \cdot [1 + (\lambda_{j,R} - \lambda_{j,s}) - m_j \cdot \delta_{j,s}]
\] (20)

We assume that \((\lambda_{j,R} - \lambda_{j,s})\) is applied when \(\lambda_{j,s} < \lambda_{j,R}\), given that \(\lambda_{j,s} < \lambda_{j,R}\) does not imply a reduction of income but a greater profitability. The income function of an investor in project \(j\) and scenario \(s\) is given by (18) and (20). Thus, the optimization problem to maximize income by the investor can be formulated as:

\[
\max_{aV_{j,s}} \sum_{s} \eta_s \cdot aV_{j,s} \cdot x_j
\]

\[
\eta_s \geq VaR_P - \sum_{j} \sum_{s} \pi_s \cdot aV_{j,s} \cdot x_j \quad \forall s \in S
\] (21)

The optimization selects the portfolio of projects, \(x_j\), in which the investor makes a bid that maximizes the overall profit assuming a risk preference. In addition, we consider an annual budget, \(\overline{aC_{j,p}}\), and a guarantee participation constraint, \(\overline{C_j}\). We assume a Scatter function with respect to the regulated income, \(SF = aV_{j,s} - aV_{j,c}\), and \(q_p\) tolerance risk level [15]. In our case, \(q_p\) is the % of the value of the portfolio without risk, that is, without cost overruns or delays in the project implementation. The probability distribution of each scenario is equally likely.

**IV. ENCOURAGING INVESTMENT AND TEP PAYMENT METHOD**

Regulation should provide mechanisms to encourage investment in grids and ensure the acceptance by agents by generating opportunities and risks. It should also define the optimal TEPs, develop it in an efficient manner and allocate costs fairly among agents using technical, economic and political criteria.

**A. Mechanisms and incentives**

There are two types of incentives. The first one, regulatory, is based on efficiency or network improvement, the second one is a market-based incentive which is associated with competition to award transmission rights, physical or financial. In general, US markets have opted for the market-based incentives and the European countries for the efficiency ones. Market experience has shown that models based on efficiency do not generate enough incentives for expansion and the market-based ones are not able to fully remunerate transmission costs. Thus, it is advisable to develop a hybrid system of incentives to obtain higher efficiency and social welfare, e.g., incentives such as increased rate of return, cost recovery, changes in capital structure, accelerated depreciation and advance income named "Construction Work In Progress-CWIP". In turn, cooperation mechanisms between the central planner and the regions, reserve corridors for the expansion plan and increase the degree of participation and compensation to the government institutions and citizens.
Two cost mechanisms are identified: socializing (all is paid) or bilateral (pay if there are benefits). The social solution promotes investment and integration of generation, but it gives an inefficient signal about the optimal location. So, a method that is socially acceptable and also efficient should be considered. To do this, two methods can be used: i) sunken cost, and ii) apparent cost. The latter cost method is divided into two additional ones: connection cost to an interconnection system (“apparent cost”), and connection cost to the main transmission system. The apparent cost approach is aimed at small generators. However, this cost does not cover the entire cost for reinforcements or expansions of the grid. That is why socialization methods are used to cover the incremental cost of transmission expansion. The sunken cost method has a disadvantage for the first investor because other investors will benefit from the free of charge grid. In the bilateral contract model, the transmission expansion is specifically designed for the beneficiaries. It should be noted that the cost may not necessarily be covered by a single contract, but several, that could include more than one user, e.g. open season mechanisms. This can be considered in 4 types of compensations: limitless auctions, ceiling price auction, auction with a ceiling/floor price, and regulated revenues (tariff mechanism).

B. Electrical behavior and bargaining issues

There are three main elements for the design of cost allocation methods. The first is that the method should reflect the short- and long-term costs. The second is that the behavior or agent response should be as expected, and, finally, that the method should be stable to market fluctuations (volatility). Thus, we consider 5 parameters for the design: i) tariff network (uni-nodal or nodal), ii) capacity (maximum design or operational), iii) agent types (generator or consumer), iv) horizon (focus on grid investment or operation system cost), and iv) indexation (fixed or variable). Therefore, it is evident that electrical methods must be harmonized with economic and social policies. International experiences show that the solutions tend to socialize allocation cost methods and proactive actions to limit the regulatory cost, i.e., among which new regulations and standards to be applied to strengthen the transmission system are designed and implemented.

C. Special Asset and Hybrid Allocation Cost Method

Economical development, technological change and sustainable policies have brought the implementation of specialized assets. Renewable Energy (RE), FACTS or reinforcement of existing assets are the assets most commonly used to respond to market evolution. Cost allocation methods have to adapt to its environment to provide efficient and fair signals. We focus on the expansion grid for integrating RE. We define a methodology that considers three elements to allocate the transmission cost of CCIS: 1) the business cost to participate in the energy market, 2) the social cost of Non-Conventional Generation (NCG), and 3) the cost of sustainability to develop a sustainable energy matrix. To determine the proportion of the business cost, $P_b$, the GGDF method is used [16]. The proportion of GNC, $P_G$, and demand, $P_S$, is determined by a bargaining model (Section III-A).

The RE stimulus arrangement considers that generation agents must meet a minimum RE quota obligation (used in Chile). We consider an incentive payment that aims at reducing the renewable transmission cost as to increase RE generation, $G_{RE}$. For this, we define a $\tau$ factor that determines the level of compliance with the quota obligation. This defines a trade-off between investment incentives in RE and bargaining with the demand to agree a re-allocation in the transmission cost and vice versa.

We assume that the bargaining process is solved by (5) and (7). The disagreement point is the value defined by the central planner/ regulator and initial GLDF/GGDF factors. The utopia point represents the non-allocation of cost. It is assumed that generators and demands have the utility function, $u(x) = 1-x$, where $x$ is a proportion of transmission cost. Once the $\tau$ factor and utility are defined, we determine the $P_G$ and $P_S$ allocation.

V. CASE STUDY

The transmission expansion game (TEG) model proposed produces the optimal value of a project. The project is part of a plan or plans of expansion defined by a central planner. The TEG framework is shown in Fig. 1. The methodology includes four main modules: 1) identifying the Elite-Pareto plans and calculation of the optimum TEP$_k$ (transmission expansion model -Section II), 2) evaluating costs and efforts according to the optimal plan to obtain the expected value, $V_e$, considering a competitive tender (cost-bid model -Section III-A/B/C), 3) once established the expected value, $V_e$, obtaining the optimal investment portfolio for the investor who has the best offers in the tender (private investment model -Section III-D), 4) determining the cost allocation between generators and demand (allocation model -Section IV). The methodology is applied to the Chilean Central Interconnected System (CCIS), modeled with a reduced transmission 55 node grid [17]. Additional features are described in [18]. The methodology described is implemented in MATLAB® 7.3 with an interface to GAMS [19]. An Intel® Core™ i5 760@ 2.80Hz processor with 6 GB of RAM is used. To calculate the optimal power flow the MATPOWER 4.0b3 tool is used [20].

A. TEP scenarios

Now, considering a constrained OO process, 2000 samples are identified with a maximum value of 363.37 MS. Twelve Pareto solutions are found, with average annual costs of 5400 MS and an average benefit (congestion cost) of 172.1 MS, when considering the multi-objective search, first with the annual operating cost and investment annuity objectives. Similarly, these solutions are considered for the methodology development. In turn, under the three objectives approach, 16 Pareto solutions are identified (Fig. 2). Regarding technological diversity, an average value of 0.32 is found. This means that diversity is 68% compared to the maximum diversity (standardized in 1). In terms of the emission values, they are between 15 and 24 MtCO$_2$. In turn, it is observed that the solutions explored consider a maximum load shedding between 350 and 620 MW. Table I shows the Elite-Pareto solutions (the top five plans). The definition of this plan considers that the lines’ capacity is below the $N-1$ criterion. In addition, if we consider a lower value for the failure cost, 0.6 $/kWh, the optimal expansion solution will be line 1-2.
**B. Optimal Cost and Bidding Competition**

For each Nash bargaining solution and investor type \((n_i=bad \text{ bidder to } n_j=good \text{ bidder})\) there is a \(RoW\) cost shown in Table II. The optimal cost of the project is determined by the cost of the asset, the effort factor and the negotiation cost of the \(RoW\). We obtain the global optimal cost expansion plan (Table II). This case shows again that in the projects where the \(RoW\) cost is not significant compared to the cost of the project, regardless of the rule or bargaining solution, the optimal cost is obtained by the desired bidder. The optimal cost \(C_j\) (that can be smaller or bigger than \(c_j\)) depends on the bidder’s type and the bargaining cost.

Table III shows the optimal cost of each bidder. It shows the cost overruns (14%) that the \(n_j\) bidder has and the efficiency "productivity" in terms of cost that the bidders \(n_1\), \(n_2\), \(n_3\), and \(n_4\) can achieve. In the case of \(n_2\) it is 11%. Table IV shows the valuation of the project based on the number of participants, \(nN\), in the auction. The results show that the winning is bidder \(n_3\) always due to the level of effort that determines a lower optimal cost. This is what the social planner seeks efficient projects with desired bidders.

**Fig. 1. Proposed methodology to transmission expansion game in CCIS.**

**Fig. 2. Non-dominated solutions for CCIS. (a) Trade-off between annual operation cost and annuity. (b) Multi-objective Pareto Solutions.**

![Table I](image)

<table>
<thead>
<tr>
<th>Plan</th>
<th>Corridor</th>
<th>Cost(M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13-14, 16-17, 17-28, 28-29, 42-45, 52-55</td>
<td>108.46</td>
</tr>
<tr>
<td>2</td>
<td>11-12, 25-27, 27-28</td>
<td>24.07</td>
</tr>
<tr>
<td>3</td>
<td>18-29, 27-28, 28-29, 28-53, 5455</td>
<td>55.14</td>
</tr>
<tr>
<td>4</td>
<td>9-10, 17-19, 24-38, 25-27, 52-53</td>
<td>166.49</td>
</tr>
<tr>
<td>5</td>
<td>9-11, 17-28, 18-29</td>
<td>144.70</td>
</tr>
<tr>
<td>Optimal</td>
<td>11-12, 25-27, 27-28 (1-2)</td>
<td>24.07 (11.65)</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Bidders</th>
<th>(R_1)</th>
<th>(R_2)</th>
<th>(R_3)</th>
<th>(R_4)</th>
<th>(R_5)</th>
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<tbody>
<tr>
<td>RoW</td>
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<td>52.9</td>
<td>58.1</td>
<td>53.2</td>
<td>57.9</td>
</tr>
<tr>
<td>(C_j)</td>
<td>786.9</td>
<td>686.6</td>
<td>663.9</td>
<td>631.5</td>
<td>630.8</td>
</tr>
</tbody>
</table>

**Table III**

<table>
<thead>
<tr>
<th>(j)</th>
<th>2N</th>
<th>3N</th>
<th>4N</th>
<th>5N</th>
<th>(C_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.9</td>
<td>19.8</td>
<td>22.3</td>
<td>23.8</td>
<td>29.8</td>
</tr>
<tr>
<td>2</td>
<td>18.3</td>
<td>24.4</td>
<td>27.4</td>
<td>29.3</td>
<td>36.6</td>
</tr>
<tr>
<td>3</td>
<td>31.9</td>
<td>42.5</td>
<td>47.9</td>
<td>51.0</td>
<td>63.8</td>
</tr>
<tr>
<td>4</td>
<td>52.3</td>
<td>69.8</td>
<td>78.5</td>
<td>83.7</td>
<td>104.7</td>
</tr>
<tr>
<td>5</td>
<td>56.5</td>
<td>75.3</td>
<td>84.7</td>
<td>90.3</td>
<td>112.9</td>
</tr>
<tr>
<td>6</td>
<td>112.6</td>
<td>150.1</td>
<td>168.9</td>
<td>180.2</td>
<td>225.2</td>
</tr>
</tbody>
</table>

**Table IV**

<table>
<thead>
<tr>
<th>(j)</th>
<th>2N</th>
<th>3N</th>
<th>4N</th>
<th>5N</th>
<th>(C_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.9</td>
<td>19.8</td>
<td>22.3</td>
<td>23.8</td>
<td>29.8</td>
</tr>
<tr>
<td>2</td>
<td>18.3</td>
<td>24.4</td>
<td>27.4</td>
<td>29.3</td>
<td>36.6</td>
</tr>
<tr>
<td>3</td>
<td>31.9</td>
<td>42.5</td>
<td>47.9</td>
<td>51.0</td>
<td>63.8</td>
</tr>
<tr>
<td>4</td>
<td>52.3</td>
<td>69.8</td>
<td>78.5</td>
<td>83.7</td>
<td>104.7</td>
</tr>
<tr>
<td>5</td>
<td>56.5</td>
<td>75.3</td>
<td>84.7</td>
<td>90.3</td>
<td>112.9</td>
</tr>
<tr>
<td>6</td>
<td>112.6</td>
<td>150.1</td>
<td>168.9</td>
<td>180.2</td>
<td>225.2</td>
</tr>
</tbody>
</table>

If the number of desired bidders is low, it is possible that a single desired bidder fails to carry out all the projects. It is therefore important to assess the increased risk on those projects where the desired bidders do not participate due to the aforementioned constraints.
C. Investment Risk

We use scenario analysis to determine the impact of risk. We consider two probability distributions: i) a \( \gamma \) distribution for the number of days of delay after the scheduled date of delivery of the project, and ii) a normal distribution to determine the value of the annual operating, maintenance and administration cost for project \( j \) and scenario \( s \), \( \lambda_{js} \). There are 5000 scenarios and the number of projects of the portfolio depends on the Elite-Pareto plan, \( EP \). The confidence level, \( \omega \), is 0.95, the discount rate, \( r \), is 10\%, and the lifetime, \( l \), is 20 years.

The normal distribution of \( \lambda_{js} \) has an average value of \( \lambda_{jR} \). The execution time of each project \( j \) is 42 months and the penalty factor for delay, \( k_p \), is 0.068\% of \( c_j \). The optimal cost, \( c_j \), the annual cost regulated rate, \( \lambda_{jR} \), and the construction time, \( t_j \) in years are shown in [18]. The number of investors participating in the tender, \( n_j \), is 10.

It is important to consider that given the discrete nature of the problem there is no curve that represents the efficient frontier. However, we identify the initial annuity portfolio without risk and the one associated with a particular annuity and \( q_p \). This defines a limited set of the portfolio, measured in terms of \( q_p \) and \( CVaR \), which define an area of risk tolerance. For example, Plan uses a dotted line to determine this area (Fig. 3). We assume that the bid is for each project and the allocation process and the beginning of the execution of the projects are in the same period of time. The optimal values, \( V_j \), for each project given the number of participants, \( n_j \), and the optimal cost of the project, \( C \), are shown in Table V.

<table>
<thead>
<tr>
<th>Project Value (( n_j = 10 ))</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_j )</td>
<td>26.82</td>
<td>32.94</td>
<td>57.42</td>
<td>94.23</td>
<td>101.61</td>
<td>202.68</td>
<td>49.96</td>
<td>14.99</td>
</tr>
</tbody>
</table>

The selection of projects takes into account the financial, construction and competition risks. In the first case, we assume a budget of M$100 and guarantees amounting to M$150. We show that, even with this budget, with a \( q_p \) higher than 7.5\% fewer projects are selected. If we consider the case of M$60 base budget and an amount for guarantees of M$100, the tolerance level decreases to 5.5\% (Fig. 3), but the selection of projects is adjusted to the new budget. This shows a trade-off between risk and budget.

![Fig. 3. Investment in CCIS expansion plan with risk tolerance \( q_p \).](image)

Another issue to consider is the effect of a higher value of \( \lambda_{jS} \). To do that we consider three scenarios: CI, the base case with \( \lambda_{jS} = \lambda_{jR} \); C2, with a value of \( \lambda_{jS} + 0.5\% \); and C3, with a value of \( \lambda_{jS} + 1\% \). We assume a constant budget of \( B_p = M$60 \). Table VI shows the variation of \( \lambda_{jS} \) does not modify \( CVaR \), but influences project selection.

<table>
<thead>
<tr>
<th>Project Selection (M$)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASE</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>CVaR</td>
<td>2.426</td>
<td>2.020</td>
<td>2.400</td>
<td>2.661</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Another important aspect to consider is the impact of delays and their respective penalties. For example, considering a doubling of the days of delays with the same % of penalty, the optimal portfolio remains the same (base case of budget = M$60) with an expected value of M$36.39. Similarly, if we change the % of penalty, the results show that the optimal portfolio does not change, just the expected value of the portfolio changes.

Additionally, we consider a case in which the central planner restricts participation on a range of projects. For example, if an agent wants to participate and make a bid for the project \( x = 3 \), is forced to bid for projects 4 and 6. This type of restrictions are applied on projects that are technologically similar and are part of an expansion corridor. Table VII shows how the selection of projects is done according to the budget constraint and the base portfolio, as well as the impact on \( CVaR \). The base case represents the case without restriction and others represent de budget constraint with a portfolio must contain if the agent wants to participate in at least one of them. A budget constraint up M$60 does not change the portfolio.

D. Encouraging Investment and TEP Payment Method

We assume that the expansion of the Pan de Azúcar-Los Vilos 500kV and Los Vilos-Polpaico 500 kV lines are for the RE integration (600 MW wind farm with 0.25 capacity factor). The base case is determined based on the study of the CNE- April 2011 [17]. The methodology shown in Fig. 1 establishes that the transmission cost allocation by wind generators does not exceed 5\%. The \( \tau \) factor is shown for an equilibrium bargaining point.

The Nash solution results tend to a socialized solution close to 40\% for NCG and 55\% for demand with a major \( \tau \) factor. In contrast, the Kalai-Smorodinsky solution defines that the maximum reduction for NCG is close to 12\% (see Fig. 4). In any case, if a NCG complies with the RE Law, it can get a reduction of its initial value. Finally, establishing the proportion, \( P_R \), makes the RE to participate in the energy market. The proportion, \( P_R \), by RE incentive and the proportion of demand, \( P_S \), that represents the cost of obtaining a sustainable matrix.
Fig. 4. Average annual allocation transmission cost by the $\tau$ factor and bargaining solutions. rNS: Nash solution. rKS: Kalai-Smorodinsky solution. a) LV-PC corresponds to the Los Vilos-Polpaico transmission asset. b) PZ-LV corresponds to the Pan de Azúcar-Los Vilos transmission asset.

VI. CONCLUSION

We have presented the application of a general transmission expansion game with several components: a TEP algorithm, bargaining solutions and hidden costs to assess a transmission project, CVaR applied to transmission investment, and a hybrid cost allocation method.

The results obtained in the test system show that the model developed is effective to solve the combinatorial problem. The multi-objective optimization under Pareto dominance and path re-liking approach defines a set of feasible solutions that establish expansion plans scenarios. A principal-agent model is used to evaluate the optimal effort of a bidder and a linear contract model to calculate the final value of a transmission project. All these features produce a realistic framework to analyze the bargaining process of a project, to create incentives for the desired bidders to be rewarded, and to maximize the social value of the project.

An investment portfolio may be different from the one established by a central planner. It is also useful for a central planner, in order to infer which projects will present a greater risk, in terms of the optimal project values and execution times. All this allows for a better criterion for the design of an efficient tender among transmission investors.

REFERENCES


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